# ECE735 Homework #1

1. Find functions $\varphi_1(x), \varphi_2(x)$ and $\varphi_3(x)$ and counting rates $\lambda_1, \lambda_2, \lambda_3$ such that is $x(0) \in \{3,7,9\}$ then for

$$dx(t) = \varphi_1(x(t))dN_1 + \varphi_2(x(t))dN_2 + \varphi_3(x(t))dN_3$$

it happens that $x(t)$ belongs to $\{3,7,9\}$ for all future time and

$$\begin{bmatrix} \dot{p}_1(t) \\ \dot{p}_2(t) \\ \dot{p}_3(t) \end{bmatrix} = \begin{bmatrix} -3 & 0 & 8 \\ 3 & -2 & 0 \\ 0 & 2 & -8 \end{bmatrix} \begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \end{bmatrix}$$

with $p_1(t)$ being the probability that $x(t)$ is 3, $p_2(t)$ being the probability that $x(t) = 7$ and $p_3(t)$ being the probability that $x(t)$ is 9.

2. N is a Poisson Counter of rate $\lambda$. Derive the Fokker-Planck equation for

$$dx = -2x\,dt - .5x dN$$

3. For the equation of exercise 2. construct a set of ordinary (deterministic) differential equations (some in $t$ and some in $\tau$) whose solution will give $\mathcal{E}x(t)x(t+\tau)$.

4. Consider a two dimensional situation in which $x(t,\tau)$ takes on the values $\{0,1,2,\ldots\}$ with $x(0,0) = 0$ and $x$ being a Poisson process of rate $\lambda$ in $t$ along lines of constant $\tau$ and a Poisson process of rate $\mu$ in $\tau$ along lines of constant $t$. Does it follow that

$$x(t,\tau) = \phi(t) \cdot \psi(\tau)$$

5. Suppose that $x(t)$ is Markovian continuous time jump process taking values in the set $X = (x_1, x_2, \ldots, x_n)$. Suppose $X = \{y_1, y_2, \ldots, y_m\}$ with $m < n$ is a second finite set and let $f : X \to Y$ be any mapping. In this case $y(t) = f(x(t))$ is called a hidden Markov model. Compute the probability distributions for the time that the $y$ processes spends in state $j$ before jumping to state $i$. Show that it is a sum of exponentials

$$q(i,j,t) = \sum_{k=1}^{r} p_{ijk} e^{-\lambda_k t}$$

6. We have taken the counting rates to be constant. If the counting rates are state dependent everything is more or less the same. Consider

$$\dot{x} = -x + z$$
$$dz = -2z\,dN$$

with the rate of $dN$ being equal to $|x|$. Show that the Itô rule is still valid with $\lambda$'s being state dependent and that

$$\frac{d}{dt}\mathcal{E}\,x = -\mathcal{E}\,x + \mathcal{E}\,|x|$$

Suppose that $x(0)$ is positive with probability one. What is the probability that $x$ stays positive forever.