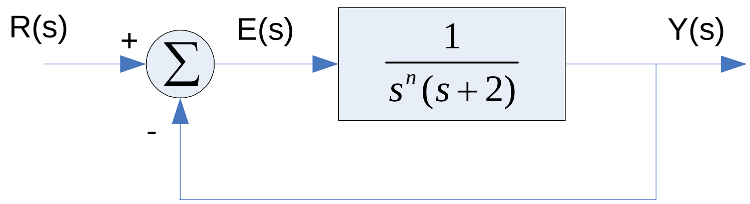
Problem 1

Consider the following feedback system:



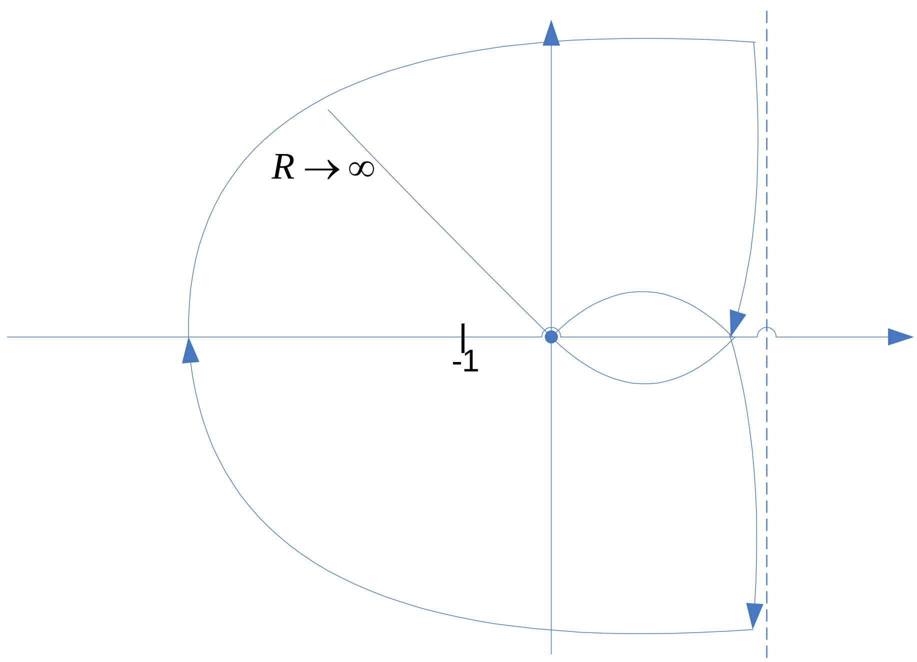
where n is positive integer, the initial conditions are zero.

a) $R(s) = \frac{1}{s^2}$, determine the steady state error when n = 1;

b) If n = 2, determine the system type and error constant of the close-loop system for reference tracking.

Problem 3

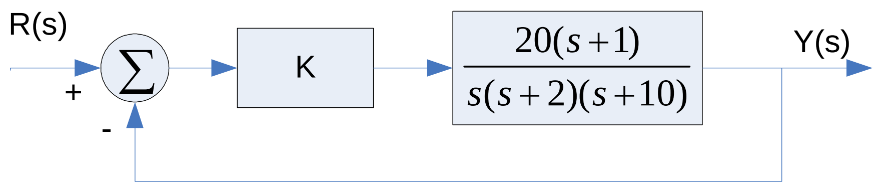
Consider the transfer function $G(s) = \frac{K(T_2 s - 1)}{s(T_1 s + 1)}$ where $K > 0$ can take different values and $T_1 > 0$, $T_2 > 0$ are fixed. The mapping of the modified Nyquist curve by G(s) and for some value of K is given below:



Is the closed loop unity feedback system with open loop transfer function G(s) stable or unstable? Can one change this condition by changing the value of K? If so, how?

Problem 4

Consider the feedback system:



1) Sketch the Bode diagrams for k=1, including asymptotes.
2) Sketch the Nyquist diagram for k=1. Determine the range of k for which system is stable.
3) Use Routh's criterion to determine the range of k for which the system is stable.

Problem 5

For the characteristic equation:

$$\Phi(s) = (s+1)(s-2) + K(s+1+3j)(s+1-3j)$$

Identify:

1) How many branches start at the poles of L(s) and how many branches end on the zeros of L(s);
2) The real axis segments of the corresponding locus;
3) The center and angles of asymptotes;
4) For what value of K and frequency are the roots on the imaginary axis;
5) Draw the root locus ($K \geq 0$) of $\Phi(s)$.

Problem 1:

$$G(s) = \frac{1}{s^n(s+2)}$$

$$H(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{1}{s^n(s+2)}}{1+\frac{1}{s^n(s+2)}} = \frac{1}{s^n(s+2)+1}$$

a). $R(s) = \frac{1}{s^2}$

$$n=1. \quad H(s) = \frac{1}{s(s+2)+1} = \frac{1}{s^2+2s+1}$$

$$E(s) = R(s) - Y(s) = R(s) - R(s)H(s) = R(s)(1-H(s))$$

$$= R(s)\left(1 - \frac{G(s)}{G(s)+1}\right) = \frac{1}{1+G(s)} R(s) = \frac{1}{1+\frac{1}{s(s+2)}} \cdot \frac{1}{s^2}$$

$$= \frac{s^2+2s}{s^2+2s+1} \cdot \frac{1}{s^2}$$

$$= \frac{s+2}{s^2+2s+1} \cdot \frac{1}{s}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \cdot \frac{s+2}{s^2+2s+1} \cdot \frac{1}{s} = \lim_{s\to 0} \frac{s+2}{s^2+2s+1} = 2$$

b) $$E(s) = \frac{1}{1+G(s)} \cdot R(s) = \frac{1}{1+\frac{1}{s^2(s+2)}} \cdot R(s)$$

$$= \frac{s^2(s+2)}{s^3+2s^2+1} \cdot R(s)$$

$$R(s) = \frac{1}{s^{k+1}}$$

$$E(s) = \frac{s^2(s+2)}{s^3+2s^2+1} \cdot \frac{1}{s^{k+1}}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s^2(s+2)}{s^3+2s^2+1} \cdot \frac{1}{s^k}$$

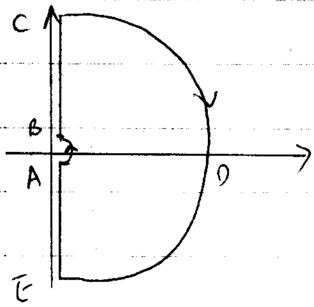
$$k=2, \quad e_{ss} = 2 = \frac{1}{K_a}$$

$\therefore$ system type $= 2$, error constant $= \frac{1}{2}$

From nyquist plot, we know $N = 1$

$G(s) = \dfrac{k(T_2 s - 1)}{s(T_1 s + 1)}$ $\therefore$ $P = 0$

$\therefore$ $Z = N + P = 1$ $\therefore$ Closed loop system is not stable.



$s = \epsilon e^{j\theta}$ $\epsilon > 0$ $\epsilon \approx 0$

from $A \to B$.

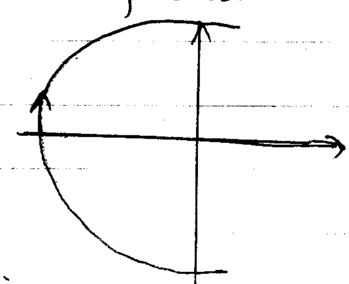
$G(s) \approx \dfrac{-k}{\epsilon e^{j\theta}}$

$\therefore$ $|G(s)| =$ infinity

$A \to B$. $s = \epsilon e^{j\theta}$ $\theta: -90^\circ \to 90^\circ$

$\angle G(s)$: $-90^\circ \to 90^\circ$

$\therefore$ Nyquist plot of $G(s)$ from $A \to B$ is the half circle around the LHP $\longrightarrow$



it does not have relationship with k.

Now, we calculate the point on the real axis.

$$G(s) = \frac{k(T_2 j\omega - 1)}{j\omega(T_1 j\omega + 1)} = \frac{k(j\omega T_2 - 1)}{-\omega^2 T_1 + j\omega} = \frac{k(j\omega T_2 - 1)(-\omega^2 T_1 - j\omega)}{(-\omega^2 T_1 + j\omega)(-\omega^2 T_1 - j\omega)}$$

$$= \frac{k(\omega^2 T_1 + \omega^2 T_2 - j\omega^3 T_1 T_2 + j\omega)}{\omega^4 T_1^2 + \omega^2 \cancel{- \omega^3 T_1 j + \omega}}$$

$\because$ the point is on the real axis

$\therefore$ $\omega - \omega^3 T_1 T_2 = 0$ $\Rightarrow$ $\omega = 0$, $\omega^2 = \dfrac{1}{T_1 T_2}$

$\omega = 0$ is the point on the origin

$\omega^2 = \dfrac{1}{T_1 T_2}$ $\therefore$ $G(s) = \dfrac{k\omega^2(T_1 + T_2)}{(\omega^2 T_1^2 + 1)\omega^2} = \dfrac{k(T_1 + T_2)}{\omega^2 T_1^2 + 1} = \dfrac{k(T_1 + T_2)}{\frac{T_1}{T_2} + 1}$

$G(s) = kT_2$ $T_2 > 0$

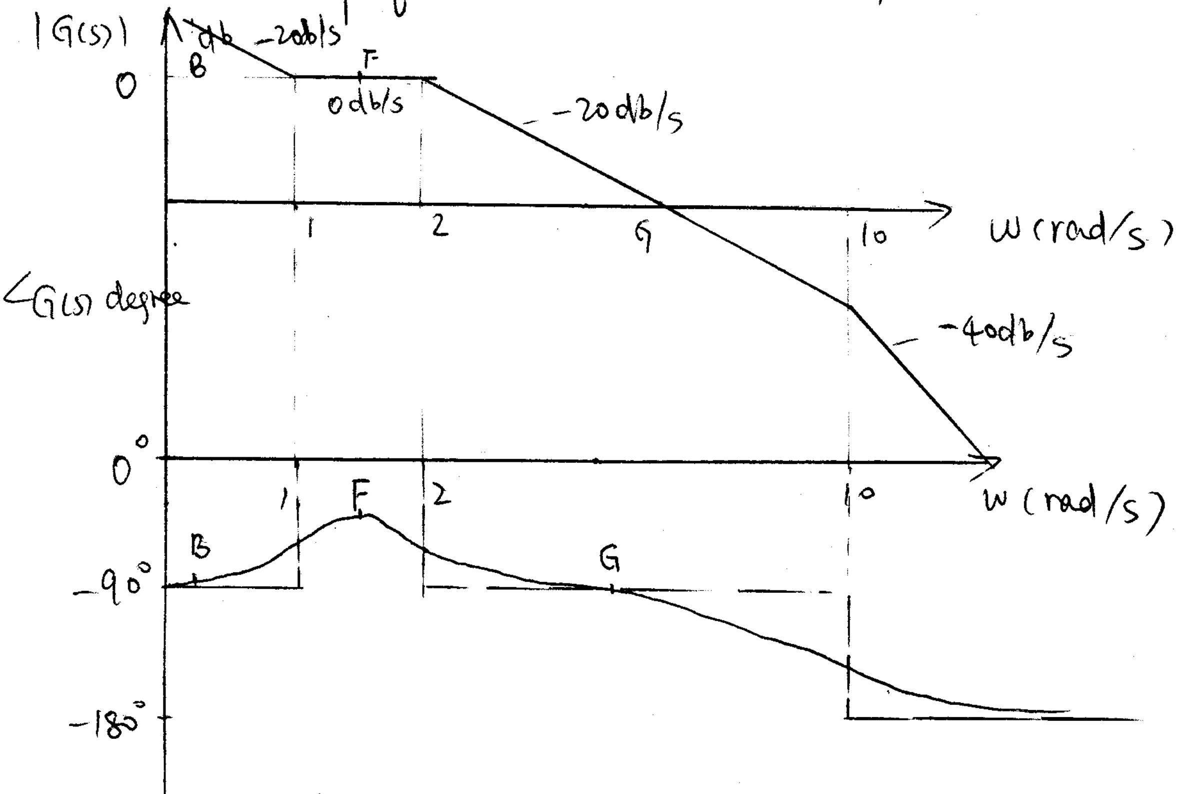
$\therefore$ change the value of k, does not change the shape of Nyquist plot of $G(s)$ $\therefore$ change the value of k won't change stability

1). $G(s) = \frac{20(s+1)}{s(s+2)(s+10)} = \frac{20(s+1)}{20s(\frac{s}{2}+1)(\frac{s}{10}+1)} = \frac{s+1}{s(\frac{s}{2}+1)(\frac{s}{10}+1)}$

Problem 4

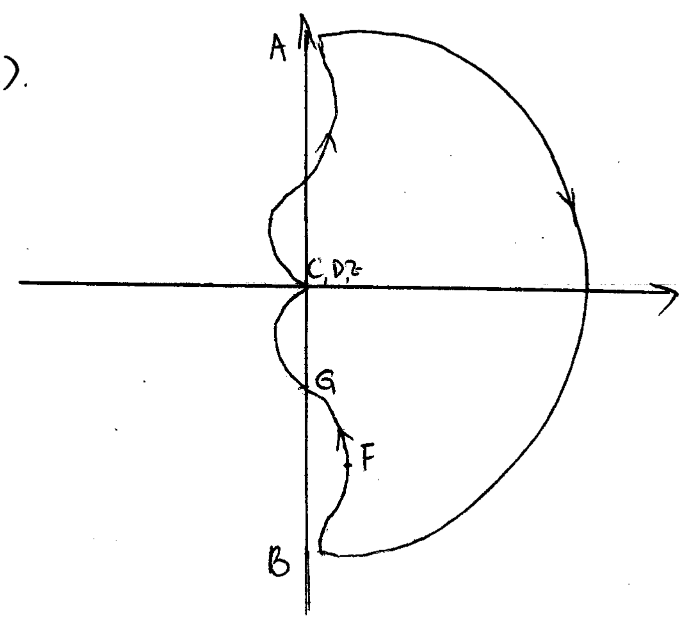
$\therefore$ $k = 1$ $20 \log k = 0$

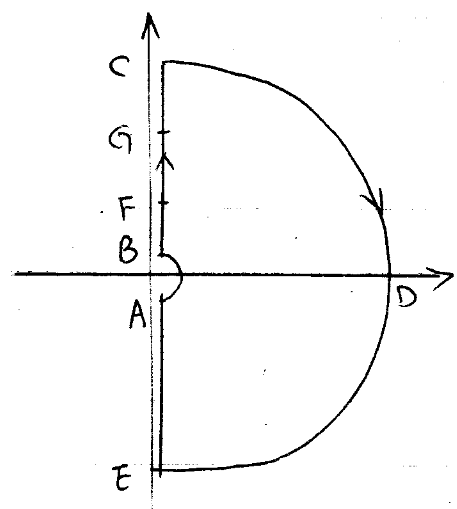
break point frequency $w = 1, 2, 10$.



From bode plot, system is stable for $k > 0$

2).





from $A \to B$ $s = \epsilon e^{j\theta}$ $\epsilon > 0$, $\epsilon \approx 0$ $\theta: -90° \to 90°$

$G(s) = \frac{20}{\epsilon e^{j\theta}}$ $|G(s)| =$ infinite   phase: $90° \to -90°$

$B \to F$ phase ↑   magnitude ↓

$F \to G$ phase ↓   magnitude ↓

From nyquist plot, system is stable for $k > 0$

3). C.F: $1+k\frac{20(s+1)}{s(s+2)(s+10)}=0$

$$s^3+12s^2+20(1+k)s+20k=0$$

s^3 1 $20(1+k)$

s^2 12 $20k$

s^1 $20(1+k)-\frac{5}{3}k$

s^0 $20k$

$\because$ system stable $\Rightarrow$ $k>0$

$20(1+k)-\frac{5}{3}k>0 \Rightarrow k>-\frac{12}{11}$ $\Big\} \Rightarrow$

$k>0$

$\therefore$ $k>0$, system is stable

Problem 5.

$$\phi(s)=(s+1)(s-2)+k(s+1+3j)(s+1-3j)$$

$a(s)=(s+1)(s-2)=s^2-s-2$

$b(s)=(s+1+3j)(s+1-3j)=s^2+2s+10$

$P_1=-1,\quad P_2=2 \qquad n=2$

$z_1=-1-3j,\quad z_2=-1+3j \qquad m=2$

1) There are 2 branches start from $s=-1$, $s=2$, end at $s=-1-3j$, $s=-1+3j$

2) The real axis segment is between $-1<s<2$

3). $n-m=0$, no asymptote.

4). arrival angle for $s=-1+3j$

the arrival angle for $s=-1-3j$ is $45°$

5) $\phi(s) = s^2 - s - 2 + ks^2 + 2ks + 10k = 0$

$(1+k)s^2 + (2k-1)s + 10k - 2 = 0$

$s^2 + \frac{2k-1}{1+k}s + \frac{10k-2}{1+k} = 0$ (1)

s^2 $\quad 1 \quad \frac{10k-2}{1+k}$

s^1 $\quad \frac{2k-1}{1+k} \quad 0$

s^0 $\quad \frac{10k-2}{1+k}$

$10k - 2 = 0 \Rightarrow k = \frac{1}{5}$

$2k - 1 = 0 \Rightarrow k = \frac{1}{2}$

$k = \frac{1}{5} \to (1) \quad 2s^2 - s = 0 \quad 2(jw)^2 - jw = 0 \Rightarrow w = 0$

$k = \frac{1}{2} \to (1) \quad \frac{3}{2}s^2 + 3 = 0 \quad \frac{3}{2}(jw)^2 + 3 = 0$

$-\frac{3}{2}w^2 + 3 = 0 \Rightarrow w^2 = 2 \quad w = \pm\sqrt{2}$

$\therefore \; k = \frac{1}{2}. \quad w = \pm\sqrt{2}$

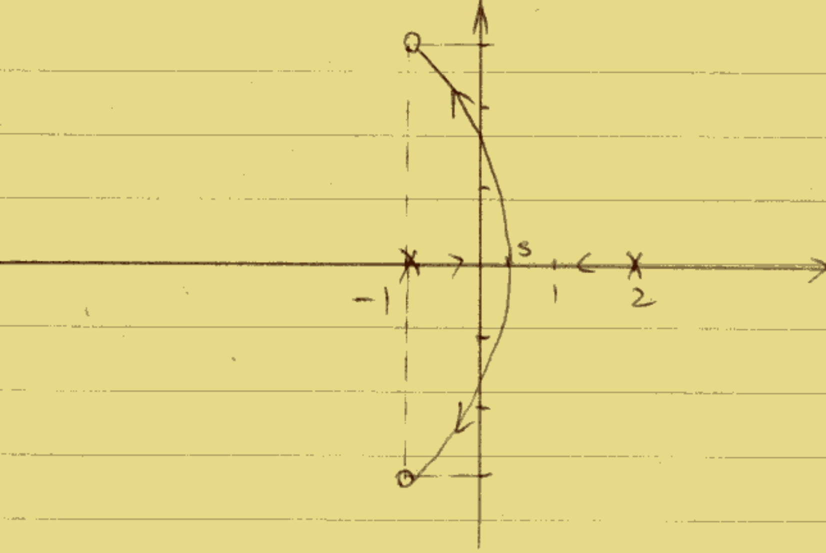
6) $b\frac{da}{ds} - a\frac{db}{ds} = 0$

$(s^2+2s+10)(2s-1) - (s^2-s-2)(2s+2) = 0$

$s^2 + 8s - 2 = 0$

$s = -4 \pm 3\sqrt{2}$

$s = -4 + 3\sqrt{2}$ is on the root locus.


s
-1
1
2