ECE 580

SOLUTIONS HW # 2

Problem 1

a) $$\frac{2}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} \qquad A = \left.\frac{2}{s+2}\right|_{s=0} = 1, \quad B = \left.\frac{2}{s}\right|_{s=-2} = -1$$

$$f(t) = \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+2}\right) = 1 - e^{-2t} \qquad t \geq 0$$

b) $$\frac{10}{s(s+1)(s+10)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+10}$$

$$A = \left.\frac{10}{(s+1)(s+10)}\right|_{s=0} = 1, \quad B = \left.\frac{10}{s(s+10)}\right|_{s=-1} = -\frac{10}{9}, \quad C = \left.\frac{10}{s(s+1)}\right|_{s=-10} = +\frac{10}{10\times 9}$$

$$f(t) = \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{\frac{10}{9}}{s+1} + \frac{\frac{1}{9}}{s+10}\right) = 1 - \frac{10}{9}e^{-t} + \frac{1}{9}e^{-10t} \qquad t \geq 0$$

g) $$\frac{s+1}{s^2} = \frac{A}{s} + \frac{B}{s^2}$$

$$B = s+1\Big|_{s=0} = 1 \qquad A = \left.\frac{d(s+1)}{ds}\right|_{s=0} = 1$$

$$f(t) = 1 + t, \quad t \geq 0$$

Problem 2

Use Laplace transforms.

$$2\ddot{x} + 7\dot{x} + 3x = 0 \qquad x(0) = 3, \quad \dot{x}(0) = 0$$

$$\Rightarrow \quad 2\left(s^2X(s) - sx(0) - \dot{x}(0)\right) + 7\left(sX(s) - x(0)\right) + 3X(s) = 0$$

$$\Rightarrow \quad X(s) = \frac{6s+21}{2s^2+7s+3} = \frac{3s+10.5}{(s+.5)(s+3)} = \frac{3.6}{s+.5} - \frac{.6}{s+3}$$

$$\Rightarrow \quad x(t) = 3.6e^{-.5t} - .6e^{-3t} \quad t \geq 0$$

Problem 3

Use Laplace Tranforms.

$$\ddot{x}(t) + \dot{x}(t) + 2.5x(t) = 0 \qquad x(0) = -1,\ \dot{x}(0) = 1$$

$$\Rightarrow \quad s^2 X(s) - s x(0) - \dot{x}(0) + sX(s) - x(0) + 2.5X(s) = 0$$

$$(s^2 + s + 2.5)X(s) = s x(0) + \dot{x}(0) + x(0)$$

$$\Rightarrow \quad X(s) = -\frac{s}{s^2+s+2.5}$$

complex conjugate roots

Now

$$X(s) = -\frac{s}{s^2+s+2.5} = \frac{-s}{(s+.5)^2+2.25} = -\left(\frac{s+.5}{(s+.5)^2+2.25} - \frac{.5}{(s+.5)^2+2.25}\right)$$

$$= -\left(\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+(\frac{3}{2})^2} - \frac{\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{2}}{(s+\frac{1}{2})^2+(\frac{3}{2})^2}\right)$$

$$= \frac{1}{3}\cdot\frac{\frac{3}{2}}{(s+\frac{1}{2})^2+(\frac{3}{2})^2} - \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+(\frac{3}{2})^2}$$

from Laplace Transform Tables

$$\Rightarrow \quad x(t) = \frac{1}{3}e^{-\frac{1}{2}t}\sin\frac{3}{2}t - e^{-\frac{1}{2}t}\cos\frac{3}{2}t \qquad 10 > t \ge 0$$

$$x(t) = \left(+\frac{1}{2} + \frac{1}{6}j\right)e^{\frac{-1+3j}{2}t} + \left(-\frac{1}{2} + \frac{1}{6}j\right)e^{\frac{-1-3j}{2}t}$$

Problem 4 (Refer to HW1 solutions).

a) $$C\frac{dV_c}{dt} + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_c + \frac{u}{R_2} = 0$$

$$y = V_c + u$$

Assume zero initial conditions and take Laplace transforms.

$$CsV_c(s) + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_c(s) = -\frac{U(s)}{R_2}$$

$$\Rightarrow \quad \left(Cs + \frac{1}{R_1} + \frac{1}{R_2}\right)V_c(s) = -\frac{U(s)}{R_2} \;\rightsquigarrow\; V_c(s) = \frac{-\frac{1}{R_2}}{Cs + \frac{1}{R_1} + \frac{1}{R_2}}\cdot U(s)$$

$$Y(s) = V_c(s) + U(s) = \frac{-\frac{1}{R_2}}{Cs + \frac{1}{R_1} + \frac{1}{R_2}}\cdot U(s) + U(s)$$

$$= \frac{Cs + \frac{1}{R_1} + \cancel{\frac{1}{R_2}} - \cancel{\frac{1}{R_2}}}{Cs + \frac{1}{R_1} + \frac{1}{R_2}}\cdot U(s) = \frac{Cs + \frac{1}{R_1}}{Cs + \frac{1}{R_1} + \frac{1}{R_2}}U(s)$$

b) $C(R_1+R_2)\frac{dv_c}{dt} + V_c = u$ $\qquad y = V_c + R_2 C\frac{dv_c}{dt}$

$$\big(C(R_1+R_2)s+1\big)V_c(s) = U(s) \qquad Y(s) = V_c(s) + R_2 C s V_c(s)$$

$$= (1+R_2Cs)V_c(s)$$

$$V_c(s) = \frac{1}{C(R_1+R_2)s+1}U(s)$$

$$\Rightarrow \quad Y(s) = \frac{1+R_2Cs}{C(R_1+R_2)s+1}U(s)$$

c) $\frac{dV_{c_1}}{dt} = V_{c_2} - 2V_1 + V_i$

$\frac{dV_{c_2}}{dt} = V_1 - V_{c_2}$ $\qquad V_0 = -V_{c_1} + V_1, \quad V_{c_2} + \frac{V_{c_2}-V_0}{R} =$

$$sV_{c_1}(s) = V_{c_2}(s) - 2\big(V_0(s)+V_{c_1}(s)\big) + V_i(s) \qquad RV_{c_2}(s) + V_{c_2}(s) = V_0$$

$$sV_{c_2}(s) = V_0(s) + V_{c_1}(s) - V_{c_2}(s) \qquad V_0(s) = (R+1)V_{c_2}$$

$$\Rightarrow \quad (s+2)V_{c_1}(s) - V_{c_2}(s) = -2V_0(s) + V_i(s)$$

mult. by s+2 and add

$$-V_{c_1}(s) + (s+1)V_{c_2}(s) = V_0(s)$$

$$-V_{c_2}(s) + \underbrace{(s+2)(s+1)}_{s^2+3s+2}V_{c_2}(s) = -2V_0(s) + V_i(s) + (s+2)V_0(s)$$

$$V_{c_2}(s)(s^2+3s+1) = sV_0(s) + V_i(s)$$

$$\Rightarrow \quad V_0(s) = (R+1)\frac{\big(sV_0(s)+V_i(s)\big)}{s^2+3s+1}$$

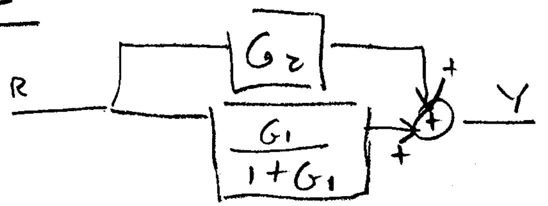
$$V_0(s)\left(1 - \frac{(R+1)s}{s^2+3s+1}\right) = \frac{(R+1)V_i(s)}{s^2+3s+1}$$

$$V_0(s)\left(\frac{s^2+3s+1-(R+1)s}{\cancel{s^2+3s+1}}\right) = \frac{(R+1)}{\cancel{s^2+3s+1}}V_i(s)$$

$$V_0(s) = \frac{R+1}{s^2+2s+1-Rs}V_i(s)$$

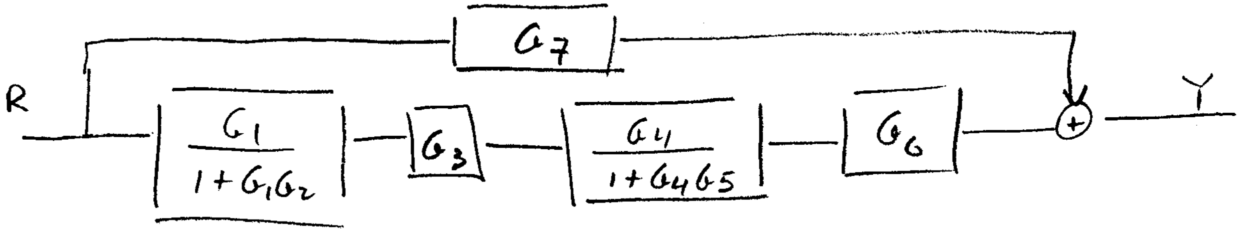
Problem 5

a)



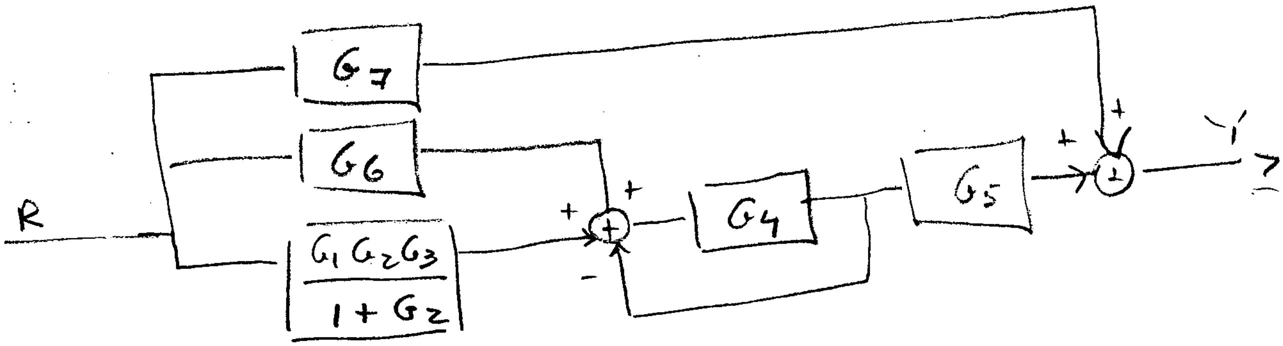
$$\sim> \quad \frac{G_1}{1+G_1} + G_2$$

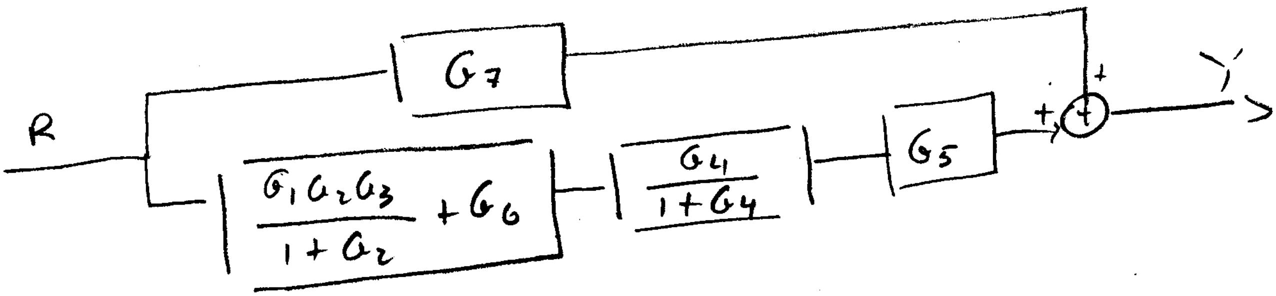
b)



$$\frac{G_1 G_3 G_4 G_6}{(1+G_1G_2)(1+G_4G_5)} + G_7$$

c)





$$G_7 + \left(\frac{G_1G_2G_3}{1+G_2} + G_6\right)\frac{G_4G_5}{1+G_4}$$