5.2. C.F. $1+K\frac{b(s)}{a(s)}=0$

1) Center of asymptote: $\alpha = \frac{\sum p_i - \sum z_i}{n-m}$

2) angle of asymptote: $\phi_l = \frac{180^\circ + 360^\circ(l-1)}{n-m}$ $l = 1, 2, \cdots, n-m$

3) departure angle: $\phi_{l,dep} = \frac{1}{q}(\sum \psi_i - \sum_{i \neq l} \phi_i - 180^\circ - 360^\circ(l-1))$

4) arrival angle: $\phi_{l,arr} = \frac{1}{q}(\sum \phi_i - \sum_{i \neq l} \psi_i + 180^\circ + 360^\circ(l-1))$

q: order of the pole or zero

ψ_i: angles from the zeros,

ϕ_i: angles from the poles.

a) $a(s) = s(s+1)$, $b(s) = s+2$

$p_1 = 0$, $p_2 = -1$; $z_1 = -2$; $n = 2$, $m = 1$, $n-m = 1$

1) $\alpha = -1+2 = 1$

2) $\phi = 180^\circ$

RULE 1. There are 2 branches to the locus.

RULE 2: The real axis segment between $-1 < s < 0$ and to the left of $s = -2$

RULE 3. There are 1 asymptotes centered at $\alpha = 1$, at the angle 180°

RULE 6:

$b = s+2$ $\frac{db}{ds} = 1$

$a = s^2+s$ $\frac{da}{ds} = 2s+1$

$b\frac{da}{ds} - a\frac{db}{ds} = (s+2)(2s+1) - (s^2+s) = 0$

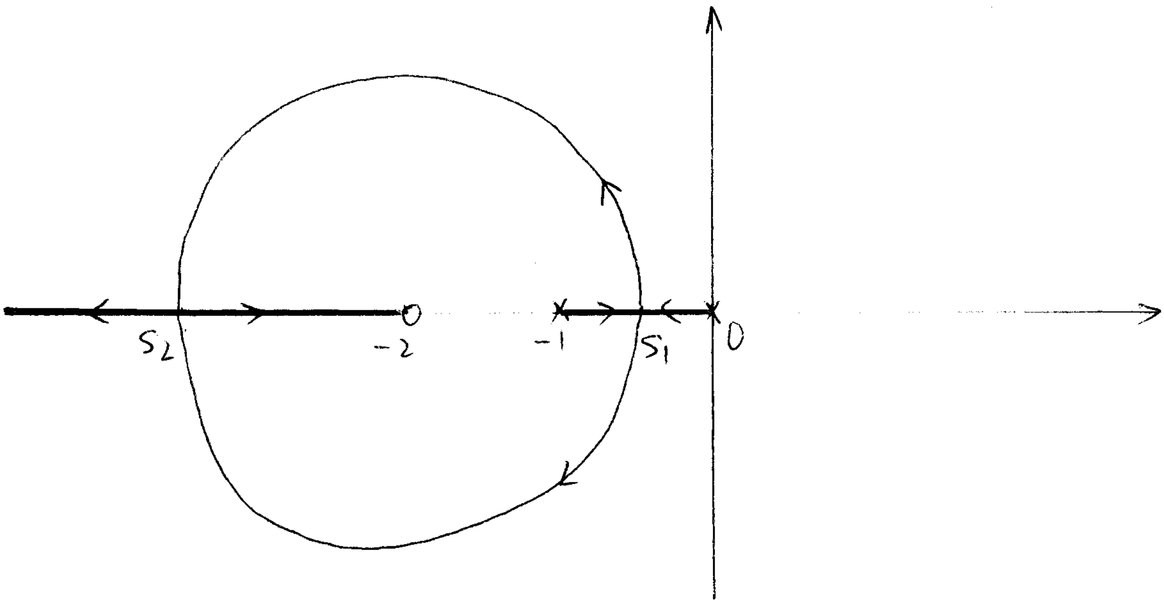
$s^2+4s+2=0 \Rightarrow s = -2 \pm \sqrt{2}$

$s_1 = -2+\sqrt{2}$, $s_2 = -2-\sqrt{2}$ are points of multiple roots.

s_2 is the break-in point and s_1 is breakaway point.

The branches depart s_1 at angles $\frac{180^\circ + 360^\circ(l-1)}{2} = \pm 90^\circ$

and will arrive s_2 at angles $\pm 90^\circ$



b) $b(s) = s+3$, $a(s) = (s+1+2j)(s+1-2j) = s^2+2s+5$

RULE 1: There are 2 branches to the locus.

RULE 2: The real axis segment is to the left of -3.

RULE 3. $P_1 = -1+2j$. $P_2 = -1-2j$ $n=2$

$z_1 = -3$ $m=1$

$\alpha = -2+3 = 1$

$\phi_l = 180^\circ$

There are 1 asymptotes centered at $\alpha = 1$ and angle is 180°

RULE 4: departure angle:

$q\,\phi_{dep} = (\Sigma\psi_i - \Sigma\phi_i - 180^\circ - 360^\circ(l-1))$

The branch departs the pole at $s = -1+2j$ at the angle

$\phi = \operatorname{arctg}\frac{2}{+2} - 90^\circ + 360^\circ(l-1) - 180^\circ$

$= +45^\circ - 90^\circ - 180^\circ = -225^\circ = 135^\circ$

The branch departs the pole at $s = -1-2j$ at the angle

$\phi = \operatorname{arctg}\frac{-2}{2} - 90^\circ - 180^\circ + 360^\circ(l-1)$

$= 135^\circ - 90^\circ - 180^\circ = -135^\circ$

RULE 6. $b(s) = s+3$ $\qquad \frac{db(s)}{ds} = 1$

$$a(s) = s^2+2s+5 \qquad \frac{da(s)}{ds} = 2s+2$$

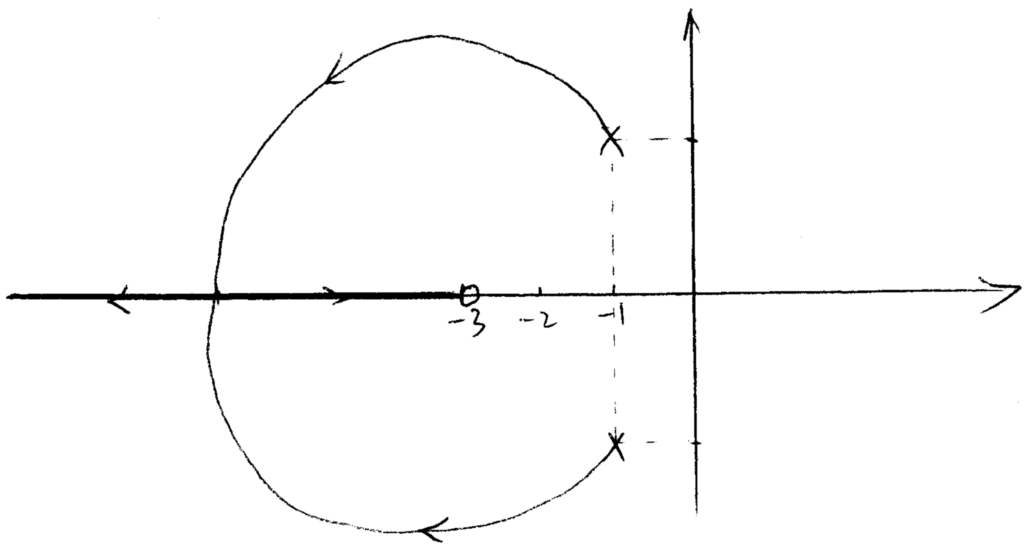
$$b(s)\frac{da(s)}{ds} - a(s)\frac{db(s)}{ds} = (s+3)(2s+2) - (s^2+2s+5) = 0$$

$$s^2+6s+1=0 \Rightarrow s = -3 \pm 2\sqrt{2}$$

$s = -3+2\sqrt{2}$ is not no the locus

$\therefore$ $s = -3-2\sqrt{2}$ is the break-in point

The branches depart $-3-2\sqrt{2}$ at angles $\pm 90°$



C). $a(s) = s^2, \quad b(s) = s+2$

$P_1 = P_2 = 0 \qquad n=2$

$z_1 = -2 \qquad m = 1$

RULE 1. There are 2 branches to the locus

RULE 2: The real axis segment is to the left of -2

RULE 3: There are 1 asymptotes centered at $\alpha = 2$ and angle is $180°$

$\alpha = 0 + 2 = 2$

$\phi_l = 180°$

RULE 4. The angles of departure from the double pole at $s=0$ are $\pm 90°$

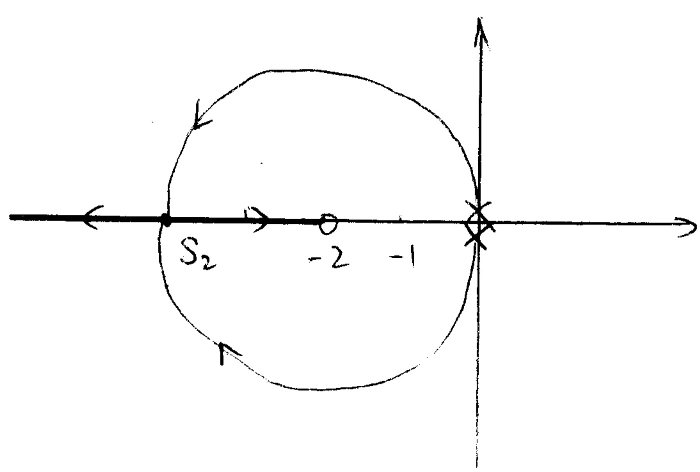
RULE 6: $a(s) = s^2 \qquad \frac{da}{ds} = 2s$

$b = s+2 \qquad \frac{db}{ds} = 1$

$b\frac{da}{ds} - a\frac{db}{ds} = (s+2)2s - s^2 = 0 \Rightarrow s^2+4s=0; \quad s_1=0, \; s_2=-4$

The point $s_2=-4$ is break-in point

The branches depart s_1 at angles $\pm 90°$ and arrive s_2 at angle $\pm 90°$



d). $a(s) = s(s+1)$; $b(s) = (s+2)(s+3)$

$P_1 = 0$, $P_2 = -1$ $n = 2$

$z_1 = -2$, $z_2 = -3$ $m = 2$

RULE 1: There are 2 branches.

RULE 2: The real axis segment is between $-2 < s < -1$ and to the left of -3.

RULE 8: $a(s) = s^2 + s$ $\frac{da}{ds} = 2s+1$

$b(s) = s^2 + 5s + 6$ $\frac{db}{ds} = 2s+5$

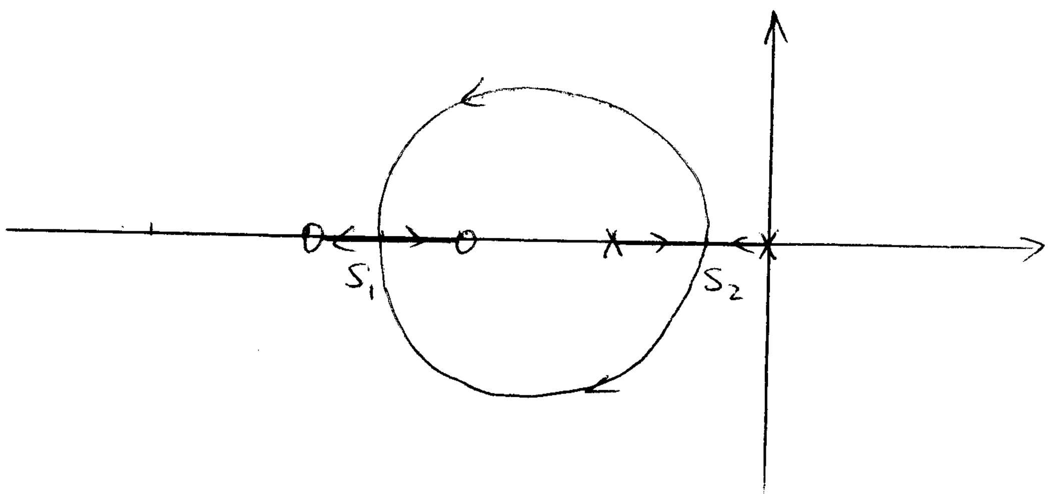
$$b\frac{da}{ds} - a\frac{db}{ds} = (s^2+5s+6)(2s+1) - (s^2+s)(2s+5) = 0$$

$$2s^2 + 6s + 3 = 0 \Rightarrow s = \frac{-3 \pm \sqrt{3}}{2}$$

$$s_1 = \frac{-3-\sqrt{3}}{2} \qquad s_2 = \frac{-3+\sqrt{3}}{2}$$

s_1 is break in point and s_2 is break-away point.

the angles for departure and arrival are $\pm 90°$



e). $a(s) = (s-1)(s+2+2j)(s+2-2j) = s^3+3s^2+4s-8$

$b(s) = 1$

$P_1 = 1$. $P_2 = -2+2j$, $P_3 = -2-2j$ $n=3$

~~$Z \neq \emptyset$~~ $m=0$

RULE 1: There are 3 branches.

RULE 2: The real axis segment is to the left of +1

RULE 3. There are 3 asymptotes

$$\alpha = \frac{1-4}{3} = -1$$

$$\phi_l = \frac{180^\circ + 360^\circ(l-1)}{3} \qquad \phi_1 = 60^\circ \qquad \phi_2 = -60^\circ \qquad \phi_3 = 180^\circ$$

RULE 4: departure angle:

$$q\phi_{dep} = \cdot/ \sum \psi_i - \sum \phi_i - 180^\circ - 360^\circ(l-1))$$

The branch departs the pole at $s=-2+2j$ at the angle.

$$q\phi = -\arctg\frac{2}{-3} - 90^\circ - 180^\circ + 360^\circ(l-1)$$

$$= -146.3^\circ - 90^\circ - 180^\circ + 360^\circ(l-1)$$

$$= -56.3^\circ$$

The branch departs the pole at $s=-2-2j$ at the angle

$$q\phi = 56.3^\circ$$

RULE 5: C.F. $s^3+3s^2+4s-8+k=0$

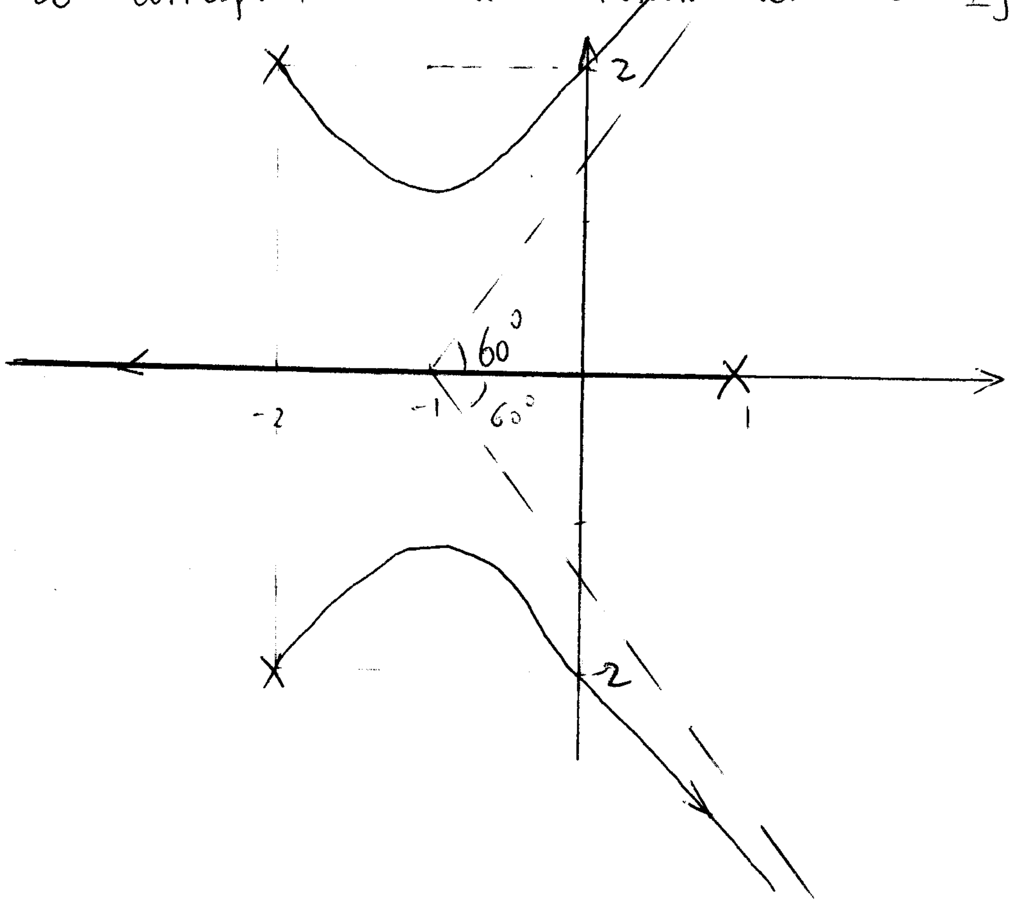
s^3	1	4
s^2	3	$k-8$
s^1	$\frac{20-k}{3}$	0
s^0	$k-8$	

$k=8$, $k=20$.

if $k=8$ $\quad (j\omega_0)^3+3(j\omega_0)^2+4(j\omega_0)=0 \Rightarrow \omega_0=0$

if $k=20$ $\quad (j\omega_0)^3+3(j\omega_0)^2+4(j\omega_0)+12=0 \Rightarrow \omega_0=\pm 2$

$\therefore$ $k=20$ correspond to a solution at $s=\pm j2$.



f) $a(s) = s^2(s+2+j)(s+2-j)(s-1) = s^5 + 3s^4 + s^3 - 5s^2$

$b(s) = s+1$

$p_1 = p_2 = 0$ $\quad p_3 = -2+j$, $\quad p_4 = -2-j$, $\quad p_5 = 1$ $\qquad n=5$

$z_1 = -1$ $\qquad m = 1$

RULE 1: There are 5 branches

RULE 2: The real axis segment is between $0 < s < 1$,

$-1 < s < 0$

RULE 3: There are 4 asymptotes

$$\alpha = \frac{-4+1+1}{4} = -0.5$$

$$\phi_l = \frac{180^\circ + 360^\circ(l-1)}{4} \qquad \phi_1 = 45^\circ,\ \phi_2 = 135^\circ,\ \phi_3 = -135^\circ,\ \phi_4 = -45$$

RULE 4: departure angle for $s = -2 \pm j$

The branch departs $s = -2+j$ at the angle

$$q\phi = \arctg\frac{1}{-1} - 2\arctg\frac{1}{-2} - \arctg\frac{1}{-3} - 90^\circ - 180^\circ + 360^\circ(l-1)$$

$$= 135^\circ - 2\times153^\circ - 161.6^\circ - 90^\circ - 180^\circ + 360^\circ(l-1)$$

$$= 117.4^\circ$$

The branch departs $s = -2-j$ at the angle -117.4°

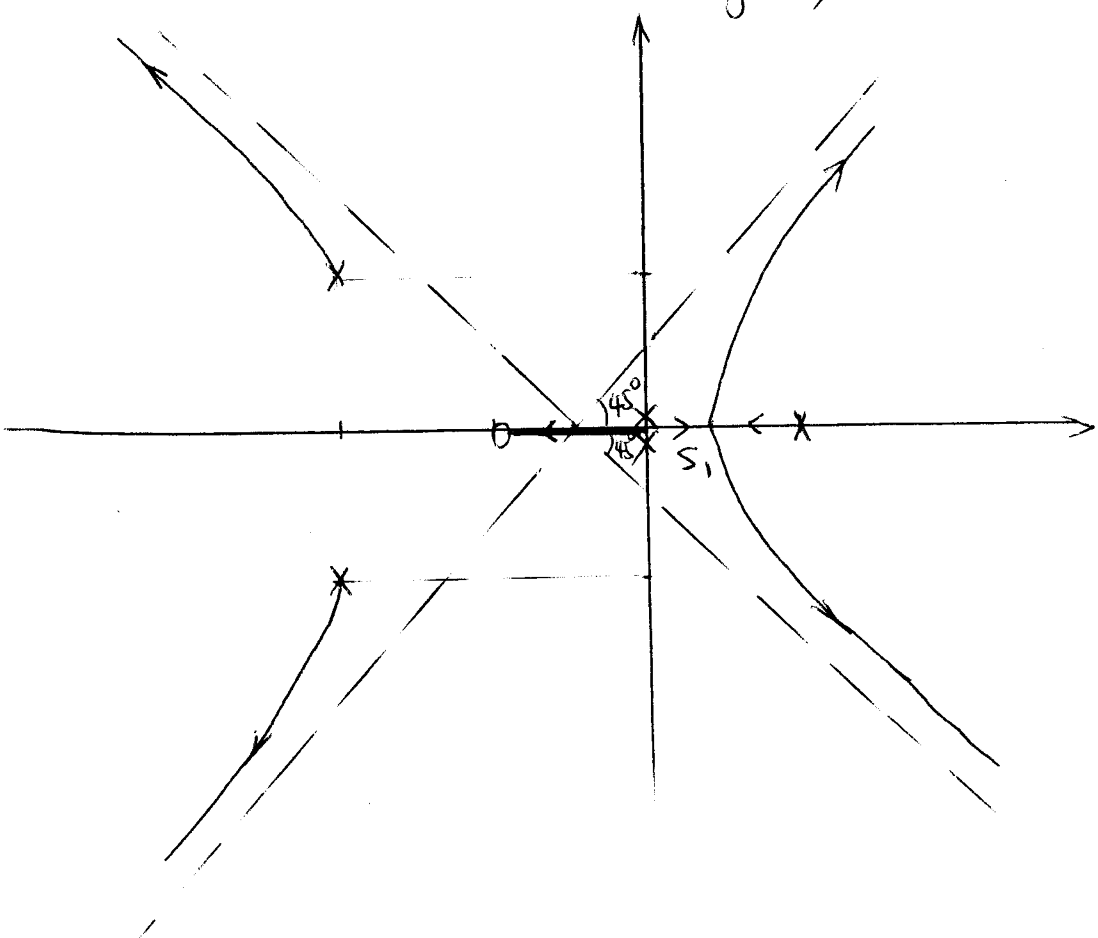
RULE 5. C.F. $s^5+3s^4+s^3-5s^2+ks+k=0$

s^5	1	1	k
s^4	3	-5	k
s^3	$\frac{8}{3}$	$+\frac{6}{5}k$	0
s^2	$-5-\frac{27}{20}k$	k	
s^1	$\frac{6}{5}k+\frac{\frac{8}{3}k}{5+\frac{27}{20}k}$		
s^0	k		

if $-5-\frac{27}{20}k=0$ or $\frac{6}{5}k+\frac{\frac{8}{3}k}{5+\frac{27}{20}k}=0$

then $k<0$

$\therefore$ the locus does not cross the imaginary axis



RULE 6.

$a(s) = s^5 + 3s^4 + s^3 - 5s^2$ $\qquad \frac{da}{ds} = 5s^4 + 12s^3 + 3s^2 - 10s$

$b(s) = s+1$ $\qquad \frac{db}{ds} = 1$

$$b\frac{da}{ds} - a\frac{db}{ds} = (s+1)(5s^4+12s^3+3s^2-10s) - (s^5+3s^4+s^3-5s^2) = 0$$

$$s^4 + 14s^3 + 14s^2 - 12s - 10 = 0$$

s_i are the root of the above polynomial.

$s_1 = -12.8418$

$s_2 = 0.8711$

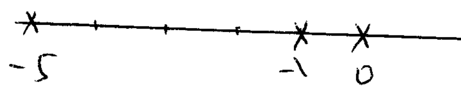
5.3. $1 + \frac{k}{s(s+1)(s+5)} = 0$

$a(s) = s(s+1)(s+5) = s^3 + 6s^2 + 5s$

$b(s) = 1$

$p_1 = -1, \quad p_2 = -5, \quad p_3 = 0 \qquad n = 3$

$\cancel{z = \emptyset} \qquad m = 0$



a). The real axis segment between $-1 < s < 0$, and to the left of -5.

b). $\alpha = \frac{-1-5+0}{3} = -2$

$\phi_l = \frac{180° + 360°(l-1)}{3} \qquad \phi_1 = 60°, \quad \phi_2 = 180°, \quad \phi_3 = -60°$

c) C.F. $s^3 + 6s^2 + 5s + k = 0$

s^3	1	5
s^2	6	k
s^1	$\frac{30-k}{6}$	0
s^0	k	

roots on the imaginary axis. $\frac{30-k}{6} = 0 \Rightarrow k = 30$

$(j\omega_0)^3 + 6(j\omega_0)^2 + 5(j\omega_0) + 30 = 0$

$-\omega_0^3 + 5\omega_0 = 0$

$-6\omega_0^2 + 30 = 0 \qquad \Rightarrow \omega_0 = \pm\sqrt{5}.$

5.4. d) $L(s) = \dfrac{(s+2)(s+4)}{s(s+1)(s+5)(s+10)}$

$a(s) = s(s+1)(s+5)(s+10) = s^4 + 16s^3 + 65s^2 + 50s$

$b(s) = (s+2)(s+4) = s^2 + 6s + 8$

$p_1 = 0, \quad p_2 = -1, \quad p_3 = -5, \quad p_4 = -10$ $\qquad n = 4$

$z_1 = -2, \quad z_2 = -4$ $\qquad m = 2.$

RULE 1: There are 4 branches

RULE 2: The real axis segment is between $-1 < s < 0$,

$-4 < s < -2, \quad -10 < s < -5$

RULE 3: There are 2 asymptotes.

$$\alpha = \frac{-1-5-10+2+4}{2} = -5$$

$$\phi_l = \frac{180^\circ + 360^\circ(l-1)}{2} \Rightarrow \phi_1 = 90^\circ, \quad \phi_2 = -90^\circ$$

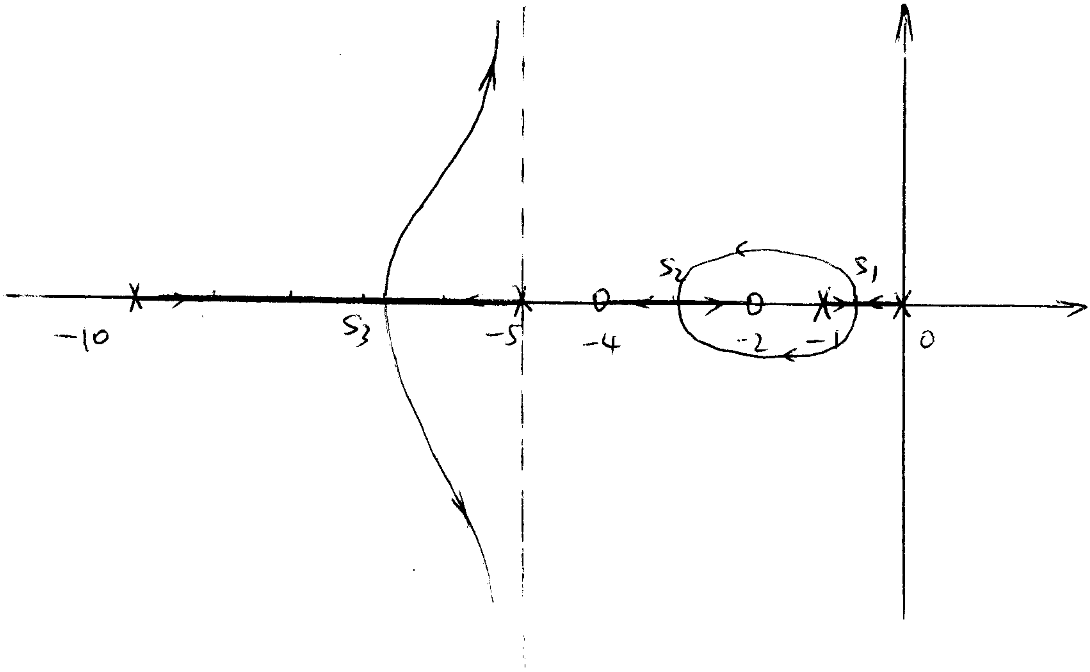
RULE 6.

$$\frac{da}{ds} = 4s^3 + 48s^2 + 130s + 50$$

$$\frac{db}{ds} = 2s + 6$$

$$b\frac{da}{ds} - a\frac{db}{ds} = (s^2+6s+8)(4s^3+48s^2+130s+50) - (s^4+16s^3+65s^2+50s)(2s+6) = 0$$

s_1, s_2, s_3 are the roots of the above polynomial.



5.5. a) $L(s) = \dfrac{1}{s^2+3s+10}$

$a(s) = s^2+3s+10$ $\qquad n=2$

$b(s) = 1$ $\qquad m=0$

$p_1 = \dfrac{-3+\sqrt{31}j}{2}$ $\qquad p_2 = \dfrac{-3-\sqrt{31}j}{2}$

RULE 1. There are 2 branches

RULE 2: There is no segment on the real axis.

RULE 3. There are 2 asymptotes

$$\alpha = \frac{-\frac{3}{2}-\frac{3}{2}}{2} = -\frac{3}{2}$$

$$\phi_l = \frac{180^\circ+360^\circ(l-1)}{2} \Rightarrow \phi_1 = 90^\circ, \quad \phi_2 = -90^\circ$$

RULE 4: departure angle

The branche departs the pole at $s = \frac{-3+\sqrt{31}j}{2}$ at the angle

$$\phi = -90^\circ - 180^\circ + 360^\circ(l-1) = 90^\circ$$

The branch departs the pole at $s = \frac{-3-\sqrt{31}j}{2}$ at the angle -90°

RULE 5. C.F. $s^2+3s+10+k=0$

s^2	1	$10+k$
s^1	3	0
s^0	$10+k$	

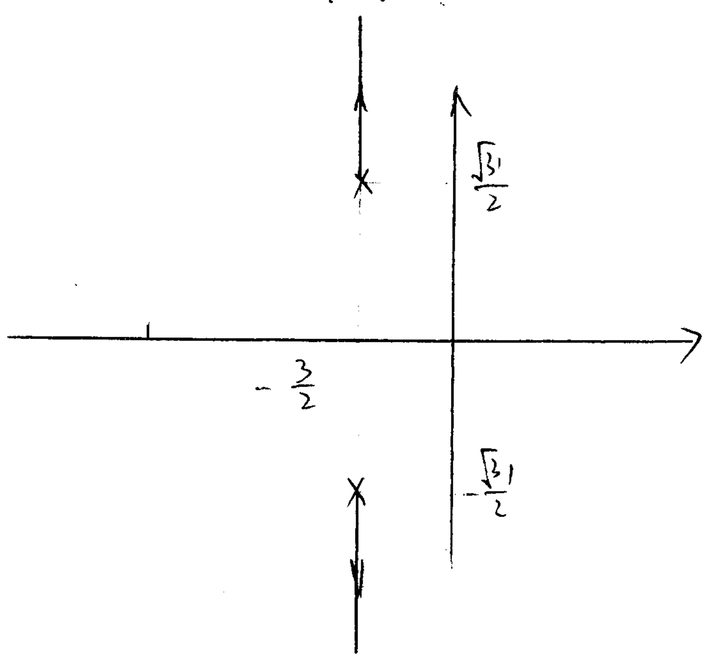
$10+k=0 \Rightarrow k=-10$

$\therefore$ there's no point on imaginary axis.

RULE 6. $\frac{da}{ds} = 2s+3$. $\frac{db}{ds} = 0$

$b\frac{da}{ds} - a\frac{db}{ds} = 0 \Rightarrow 2s+3=0 \Rightarrow s=-3$

$s=-3$ is not on locus.



f). $L(s) = \dfrac{s^2+4}{s(s^2+1)}$

$a(s) = s(s^2+1) = s^3+s$

$b(s) = s^2+4$

$p_1 = 0, \quad p_2 = \sqrt{-1} = j \quad p_3 = -j$ $\qquad n=3$

$z_1 = -2j, \quad z_2 = 2j$ $\qquad m=2$

RULE 1. There are 3 branches

RULE 2: There ~~is~~ ~~no~~ segment on the real axis, to the left of 0

RULE 3: There is 1 asymptote

$\alpha = 0$

$\phi_l = \dfrac{180^\circ + 360^\circ(l-1)}{3-2} \Rightarrow \phi_1 = 180^\circ.$

RULE 4. departure angle.

The branch departs the pole at $s=j$ at the angle:

$q\phi_d = -90^\circ + 90^\circ - 90^\circ - 90^\circ - 180^\circ + 360^\circ(l-1) = 0^\circ$

The branch departs the pole at $s=-j$ at the angle 0°

arrival angle.

The branch arrives the zero at $s=2j$ at the angle.

$q\phi_a = 90^\circ + 90^\circ + 90^\circ - 90^\circ + 180^\circ + 360^\circ(l-1) = 0^\circ$

The branch arrives the zero at $s=-2j$ at the angle 0°

RULE 5. C.F. $s^3+s+ks^2+4k=0$

$$s^3+ks^2+s+4k=0$$

s^3	1	1
s^2	k	4k
s^1	-3	0
s^0	4k	

$k=0$

There's no point on imaginary axis.