

18. (a) Sketch the Nyquist plot for an open-loop system with transfer function $1/s^2$; that is, sketch

$$\frac{1}{s^2} \Big|_{s=C_1},$$

where C_1 is a contour enclosing the entire RHP, as shown in Fig. ??.
 (Hint: Assume C_1 takes a small detour around the poles at $s = 0$, as shown in Fig. ??.)

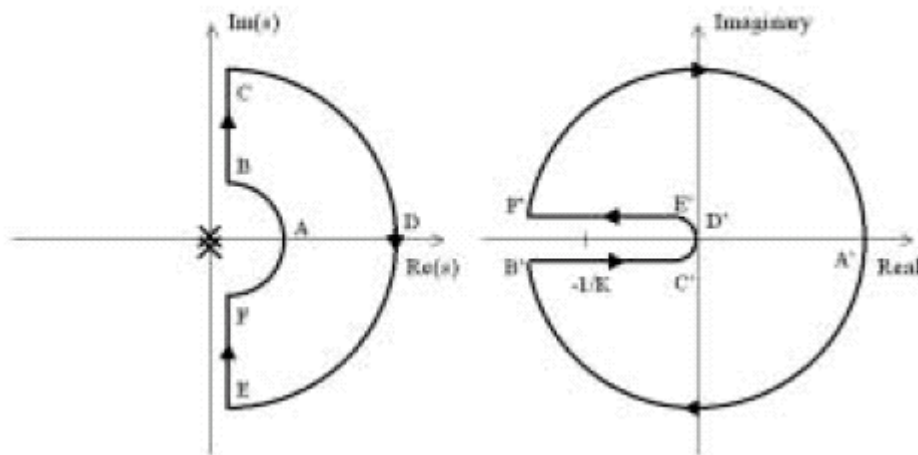
- (b) Repeat part (a) for an open-loop system whose transfer function is $G(s) = 1/(s^2 + \omega_0^2)$.

Solution :

- (a)

$$G(s) = \frac{1}{s^2}$$

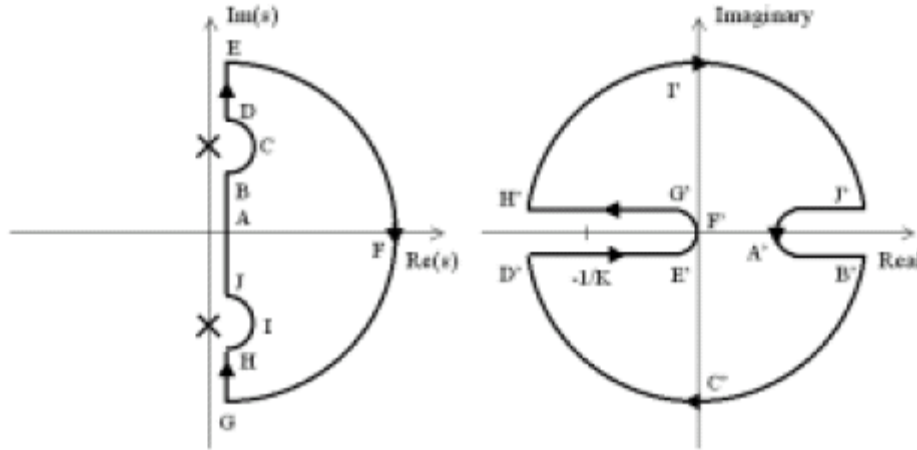
Note that the portion of the Nyquist plot on the right side below that corresponds to the bode plot is from B' to C'. The large loop from F' to A' to B' arises from the detour around the 2 poles at the origin.



- (b)

$$G(s) = \frac{1}{s^2 + \omega_0^2}$$

Note here that the portion of the Nyquist plot coming directly from a Bode plot is the portion from A' to E'. That portion includes a 180° arc that arose because of the detour around the pole on the imaginary axis.



1) From $A \rightarrow B$, set $s = j\omega$, $G(s) = \frac{1}{j\omega - j\omega_0} \frac{1}{j\omega + j\omega_0}$

$\because \omega < \omega_0$, $\therefore \phi = 90^\circ - 90^\circ = 0^\circ$ the phase of $G(s)$ from $A \rightarrow B$ are all 0° , however, the magnitude of $G(s)$ is decreasing.

2) For the half circle $B \rightarrow C \rightarrow D$, we set $s = j\omega_0 + \varepsilon e^{j\theta}$, $\varepsilon > 0$ and $\varepsilon \approx 0$.

Then $G(s) = \frac{1}{2j\omega_0 + \varepsilon e^{j\theta}} \frac{1}{\varepsilon e^{j\theta}}$, because $\varepsilon \approx 0$,

$$G(s) \approx \frac{1}{2j\omega_0} \frac{1}{\varepsilon e^{j\theta}}$$

$$|G(s)| = \infty$$

$$\phi = -90^\circ - \theta \quad \text{Phase of } G(s)$$

for point B: $\theta = -90^\circ$, $\therefore \phi = -90^\circ - \theta = 0^\circ$

for point C: $\theta = 0^\circ$, $\therefore \phi = -90^\circ - \theta = -90^\circ$

for point D: $\theta = +90^\circ$, $\therefore \phi = -90^\circ - \theta = -180^\circ$

so the phase of $G(s)$ goes from $0^\circ \rightarrow -90^\circ \rightarrow -180^\circ$, the phase is infinity (Shown in the right figure, from $B' \rightarrow C' \rightarrow D'$)

3) From $D \rightarrow E$, set $s = j\omega$, $G(s) = \frac{1}{j\omega - j\omega_0} \frac{1}{j\omega + j\omega_0}$

$\because \omega > \omega_0$, $\therefore \phi = -90^\circ - 90^\circ = -180^\circ$ the phase of $G(s)$ from $A \rightarrow B$ are all -180° , however, the magnitude of $G(s)$ is increasing.

The Nyquist plot is showed above in the right figure.

19. Sketch the Nyquist plot based on the Bode plots for each of the following systems, then compare your result with that obtained using the MATLAB command `nyquist`:

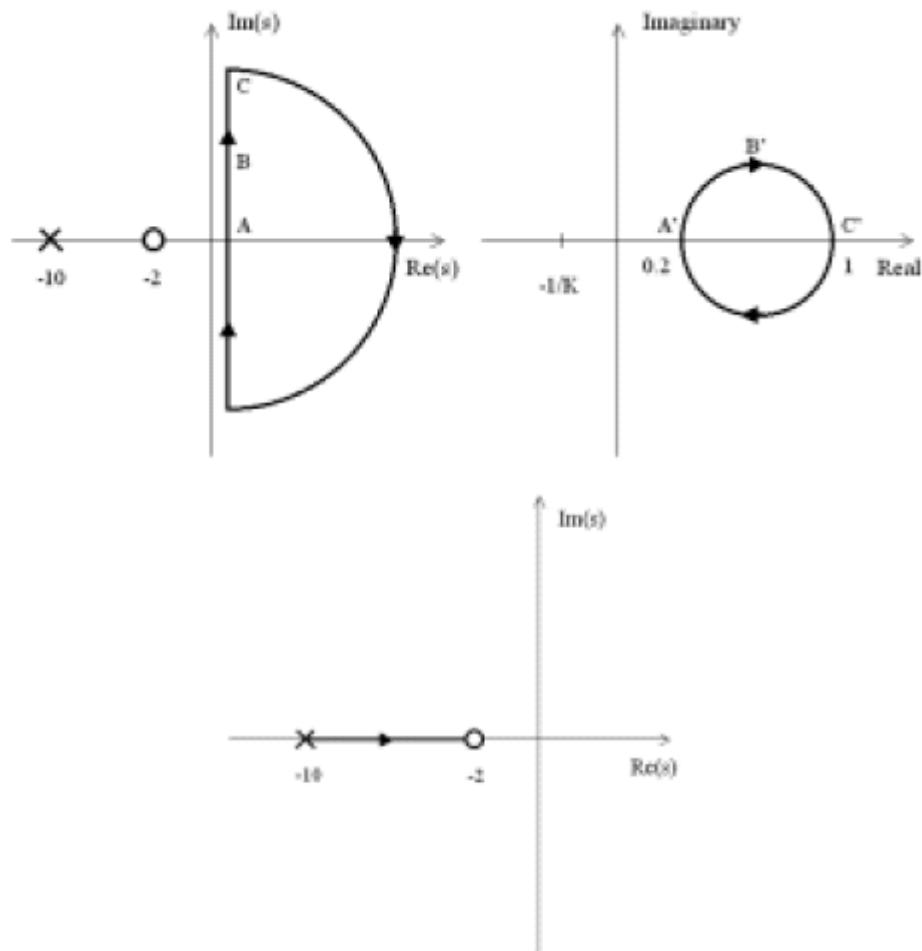
(a) $KG(s) = \frac{K(s+2)}{s+10}$

(b) $KG(s) = \frac{K}{(s+10)(s+2)^2}$

(c) $KG(s) = \frac{K(s+10)(s+1)}{(s+100)(s+2)^3}$

Solution :

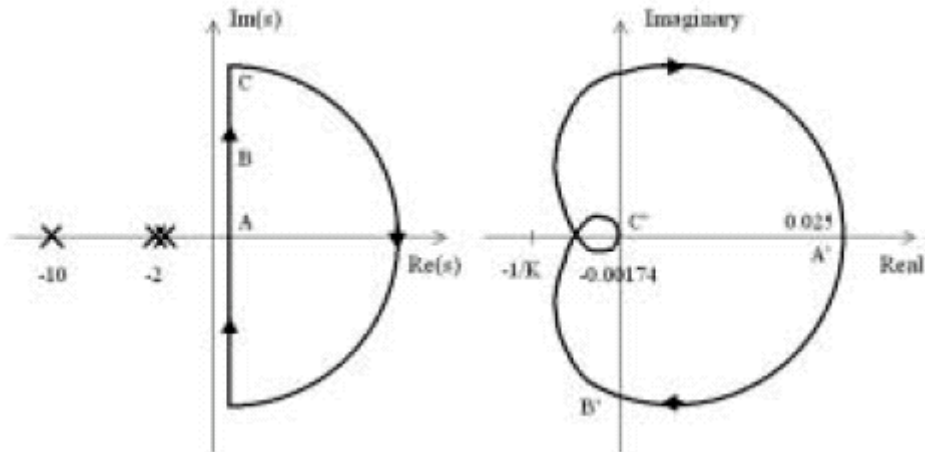
(a)



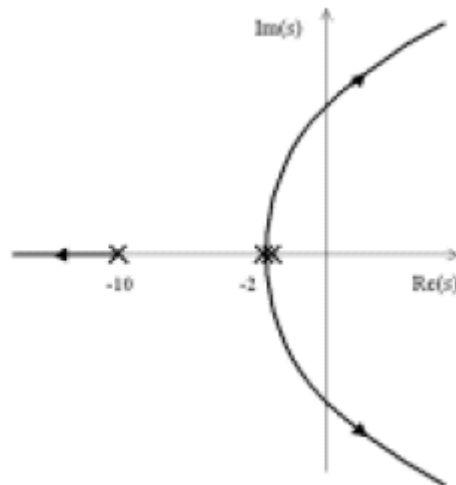
$$N = 0, P = 0 \implies Z = N + P = 0$$

The closed-loop system is stable for any $K > 0$.

- (b) The Bode plot shows an initial phase of 0° hence the Nyquist starts on the positive real axis at A' . The Bode ends with a phase of -270° hence the Nyquist ends the bottom loop by approaching the origin from the positive imaginary axis (or an angle of -270°).

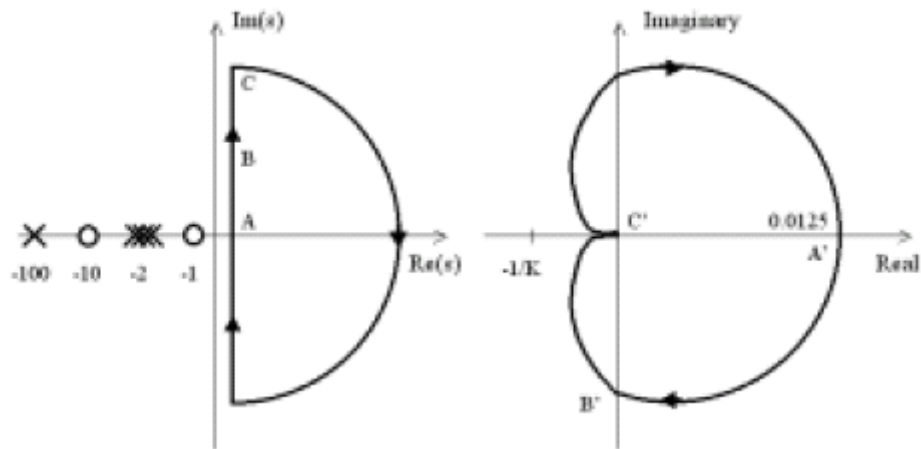


The magnitude of the Nyquist plot as it crosses the negative real axis is 0.00174 . It will not encircle the $-1/K$ point until $K = 1/0.00174 = 576$.

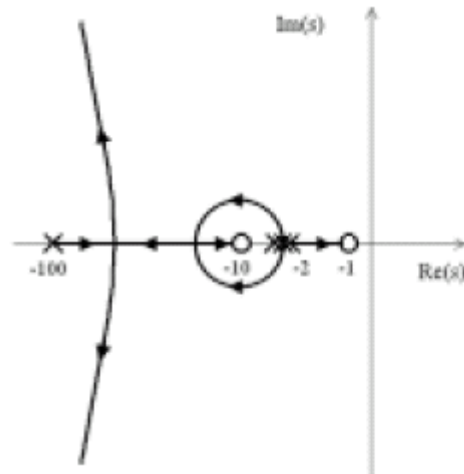


- i. $0 < K < 576$
 $N = 0, P = 0 \implies Z = N + P = 0$
 The closed-loop system is stable.
 - ii. $K > 576$
 $N = 2, P = 0 \implies Z = N + P = 2$
 The closed-loop system has two unstable roots as verified by the root locus.
- (c) The Bode plot shows an initial phase of 0° hence the Nyquist starts on the positive real axis at A' . The Bode ends with a phase of -180° hence the Nyquist ends the bottom loop by approaching the

origin from the negative real axis (or an angle of -180°).



It will never encircle the $-1/K$ point, hence it is always stable. The root locus below confirms that.



$$N = 0, P = 0 \implies Z = N + P = 0$$

The closed-loop system is stable for any $K > 0$.

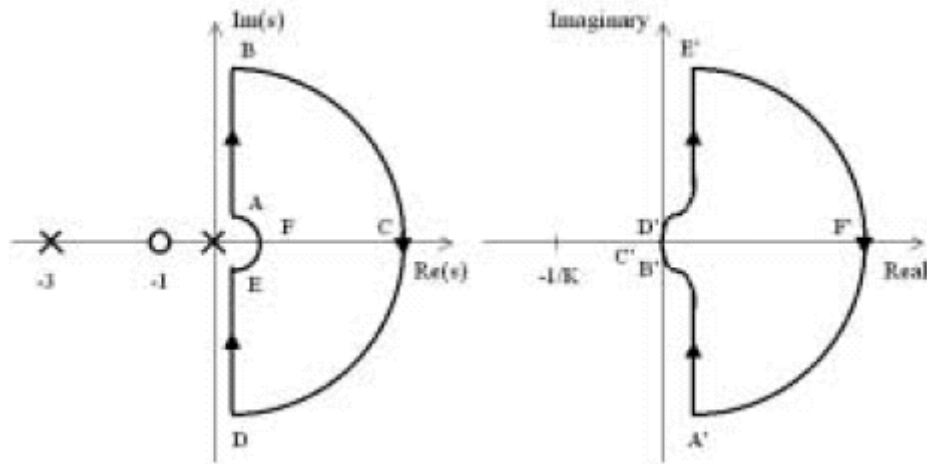
20. Draw a Nyquist plot for

$$KG(s) = \frac{K(s+1)}{s(s+3)} \quad (1)$$

choosing the contour to be to the right of the singularity on the $j\omega$ -axis, and determine the range of K for which the system is stable using the Nyquist Criterion. Then redo the Nyquist plot, this time choosing the contour to be to the left of the singularity on the imaginary axis and again check the range of K for which the system is stable using the Nyquist Criterion. Are the answers the same? Should they be?

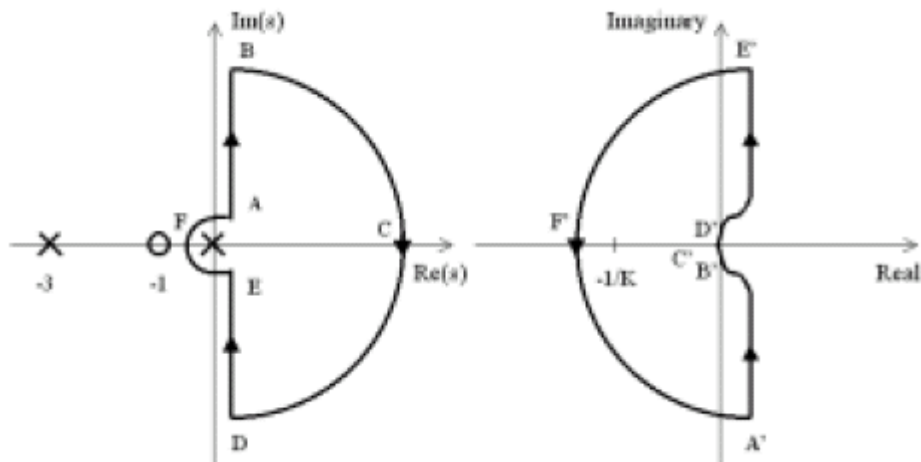
Solution :

If you choose the contour to the right of the singularity on the origin, the Nyquist plot looks like this :



From the Nyquist plot, the range of K for stability is $-\frac{1}{K} < 0$ ($N = 0, P = 0 \implies Z = N + P = 0$). So the system is stable for $K > 0$.

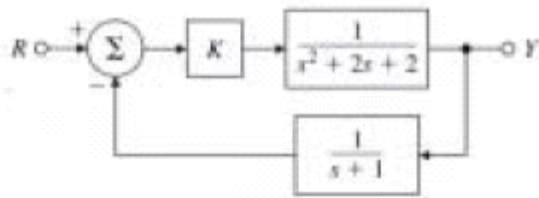
Similarly, in the case with the contour to the left of the singularity on the origin, the Nyquist plot is:



From the Nyquist plot, the range of K for stability is $-\frac{1}{K} < 0$ ($N = -1, P = 1 \implies Z = N + P = 0$). So the system is stable for $K > 0$.

The way of choosing the contour around singularity on the $j\omega$ -axis does not affect its stability criterion. The results should be the same in either way. However, it is somewhat less cumbersome to pick the contour to the right of a pole on the imaginary axis so that there are no unstable poles within the contour, hence $P=0$.

Figure 6.91: Control system for Problem 21



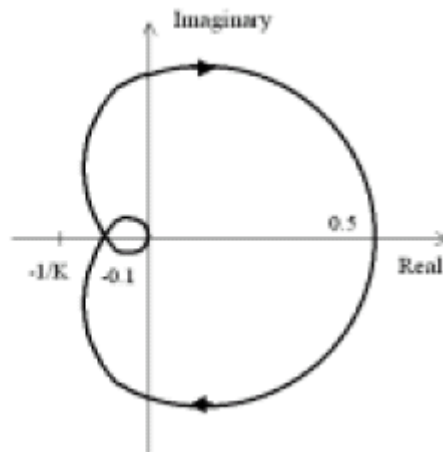
21. Draw the Nyquist plot for the system in Fig. 6.91. Using the Nyquist stability criterion, determine the range of K for which the system is stable. Consider both positive and negative values of K .

Solution :

The characteristic equation:

$$1 + K \frac{1}{(s^2 + 2s + 2)} \frac{1}{(s + 1)} = 0$$

$$G(s) = \frac{1}{(s + 1)(s^2 + 2s + 2)}$$



set $s = j\omega$

$$\begin{aligned} G(s) &= \frac{1}{(j\omega + 1)[(j\omega)^2 + 2j\omega + 2]} \\ &= \frac{1}{(j\omega + 1)(2 - \omega^2 + 2j\omega)} \\ &= \frac{1}{2 - 3\omega^2 + j(4\omega - \omega^3)} \quad (1) \end{aligned}$$

For the points on the real axis, $\text{Im}(G(s))=0$, $\therefore 4\omega - \omega^3 = 0 \Rightarrow \omega = 0$ or $\omega = \pm 2$

when $\omega = 0$, substitute into (1), $G(s) = 0.5$

when $\omega = \pm 2$, substitute into (1), $G(s) = -0.1$

if $K > 0$, then $-1/K < -0.1 \Rightarrow 0 < K < 10$

if $K < 0$, the entire nyquist plot is essentially flipped about the imaginary axis,

so $-1/K > 0.5 \Rightarrow -2 < K < 0$

therefore, the range of K for stability is $-2 < K < 10$

For positive K , note that the magnitude of the Nyquist plot as it crosses the negative real axis is 0.1, hence $K < 10$ for stability. For negative K , the entire Nyquist plot is essentially flipped about the imaginary axis, thus the magnitude where it crosses the negative real axis will be 0.5 and the stability limit is that $|K| < 2$. Therefore, the range of K for stability is $-2 < K < 10$.