8. Consider the satellite-attitude control problem shown in Fig. 4.40 where the normalized parameters are

$J = 10$ spacecraft inertia, N-m-sec$^2/rad$
$\theta_r =$ reference satellite attitude, rad.
$\theta =$ actual satellite attitude, rad.
$H_y = 1$ sensor scale, factor volts/rad.
$H_r = 1$ reference sensor scale factor, volts/rad.
$w =$ disturbance torque. N-m

(a) Use proportional control, P, with $D(s) = k_p$, and give the range of values for k_p for which the system will be stable.

(b) Use PD control and let $D(s) = (k_p + k_D s)$ and determine the system type and error constant with respect to reference inputs.

(c) Use PD control, let $D(s) = (k_p + k_D s)$ and determine the system type and error constant with respect to disturbance inputs.

(d) Use PI control, let $D(s) = (k_p + k_I/s)$, and determine the system type and error constant with respect to reference inputs.

(e) Use PI control, let $D(s) = (k_p + k_I/s)$, and determine the system type and error constant with respect to disturbance inputs.

(f) Use PID control, let $D(s) = (k_p + k_I/s + k_D s)$ and determine the system type and error constant with respect to reference inputs.

(g) Use PID control, let $D(s) = (k_p + k_I/s + k_D s)$ and determine the system type and error constant with respect to disturbance inputs.

Solution:

(a) $D(s) = k_p$; The characteristic equation is

$$1 + H_y D(s)\frac{1}{Js^2} = 0$$

$$Js^2 + H_y k_p = 0$$

or $s = \pm j\sqrt{\frac{H_y k_p}{J}}$ so that no additional damping is provided. The system cannot be made stable with proportional control alone.

(b) Steady-state error to reference steps.

$$\frac{\Theta(s)}{\Theta_r(s)} = H_r \frac{D(s)\frac{1}{Js^2}}{1 + D(s)H_y\frac{1}{Js^2}}$$

$$=H_r\frac{(k_p+k_Ds)}{Js^2+(k_p+k_Ds)H_y}$$

$$E(s)=\theta_r(s)-\theta(s)=(1-\frac{k_p+k_Ds}{Js^2+k_Ds+k_p})\theta_r(s)=\frac{Js^2}{Js^2+k_Ds+k_p}\theta_r(s)$$

Assuming the system is stable

$$e_{ss}=\lim_{s\to o}sE(s)=\lim_{s\to o}s\frac{Js^2}{Js^2+k_Ds+k_p}\theta_r(s)=\lim_{s\to o}s\frac{Js^2}{Js^2+k_Ds+k_p}\frac{1}{s^{k+1}}$$

$$e_{ss}=\lim_{s\to o}\frac{Js^2}{Js^2+k_Ds+k_p}\frac{1}{s^k}$$

$$k=2,\ e_{ss}=\frac{J}{k_p}$$

therefore the system is type 2 and acceleration constant is $K_v=\frac{k_p}{J}=\frac{k_p}{10}$

(c) Steady-state error to disturbance steps

$$\frac{\Theta(s)}{W(s)}=\frac{1}{Js^2+(k_p+k_Ds)H_y}$$

$$\frac{E(s)}{W(s)}=-\frac{\theta(s)}{W(s)}=\frac{-1}{Js^2+k_Ds+k_p}=T_w(s)=s^0T_{0,w}(s)$$

Assuming the system is stable

$$y_{ss}=\lim_{s\to 0}sT_w(s)\frac{1}{s^{k+1}}=\lim_{s\to 0}T_w(s)\frac{1}{s^k}=\lim_{s\to 0}T_{0,w}(s)\frac{s^0}{s^k}$$

$$n=0,\ k=0,\ y_{ss}=\frac{-1}{k_p}$$

therefore the system is type 0 and error constant is $-k_p$

(d) The characteristic equation is

$$1+H_yD(s)\frac{1}{Js^2}=0$$

With PI control,

$$Js^3+H_yk_ps+H_yk_I=0$$

From the Routh's stability criterion, the system will always have (at least) one pole not in the LHP. Hence, this is not a good control strategy. If all poles of sY(s) are in the left half of s-plane, $\lim_{t\to\infty}y(t)=\lim_{s\to 0}sY(s)$. We cannot use FVT here.

(e) see d bove.

(f) The characteristic equation with PID control is

$$1+H_y(k_p+\frac{k_I}{s}+k_Ds)\frac{1}{Js^2}=0$$

$$\frac{\Theta(s)}{\Theta_r(s)} = H_r\frac{D(s)\frac{1}{Js^2}}{1+D(s)H_y\frac{1}{Js^2}}$$

$$= \frac{H_r(k_p+\frac{k_I}{s}+k_Ds)}{Js^2+(k_p+\frac{k_I}{s}+k_Ds)H_y}$$

$$= \frac{H_r(k_Ds^2+k_ps+k_I)}{Js^3+(k_Ds^2+k_ps+k_I)H_y}$$

or

$$Js^3+H_yk_Ds^2+H_yk_ps+H_yk_I=0$$

There is now control over all the three poles and the system can be made stable.

$$E(s)=\theta_r(s)-\theta(s)=(1-\frac{k_Ds^2+k_ps+k_I}{Js^3+k_Ds^2+k_ps+k_I})\theta_r(s)=\frac{Js^3}{Js^3+k_Ds^2+k_ps+k_I}\theta_r(s)$$

Assuming the system is stable

$$e_{ss}=\lim_{s\to o}sE(s)=\lim_{s\to o}s\frac{Js^3}{Js^3+k_Ds^2+k_ps+k_I}\theta_r(s)=\lim_{s\to o}s\frac{Js^3}{Js^3+k_Ds^2+k_ps+k_I}\frac{1}{s^{k+1}}$$

$$e_{ss}=\lim_{s\to o}\frac{Js^3}{Js^3+k_Ds^2+k_ps+k_I}\frac{1}{s^k}$$

$$k=3,\ e_{ss}=\frac{J}{k_I}$$

therefore the system is type 3 and error constant is $\frac{k_I}{J}=\frac{k_I}{10}$

(g) The error to a disturbance is found from

$$\frac{\Theta(s)}{W(s)}=\frac{s}{Js^3+H_y(k_Ds^2+k_ps+k_I)}$$

$$\frac{E(s)}{W(s)}=-\frac{\theta(s)}{W(s)}=\frac{-s}{Js^3+k_Ds^2+k_ps+k_I}=T_w(s)=s^1T_{0,w}(s)$$

$$T_{0,w}(s)=\frac{-1}{Js^3+k_Ds^2+k_ps+k_I}$$

Assuming the system is stable

$$y_{ss}=\lim_{s\to 0}sT_w(s)\frac{1}{s^{k+1}}=\lim_{s\to 0}T_w(s)\frac{1}{s^k}=\lim_{s\to 0}T_{0,w}(s)\frac{s^1}{s^k}$$

$$n=1,\ k=1,\ y_{ss}=\frac{-1}{k_I}$$

therefore the system is type 1 and error constant is $-k_I$

12. Consider the second-order plant

$$G(s) = \frac{1}{(s+1)(5s+1)}.$$

(a) Determine the system type and error constant with respect to tracking polynomial reference inputs of the system for P, PD, and PID controllers (as configured in Fig.4.2(b)) Let $k_p = 19$, $k_I = 9.5$, and $k_D = 4$.

(b) Determine the system type and error constant of the system with respect to disturbance inputs for each of the three regulators in part (a) with respect to rejecting polynomial disturbances $w(t)$ at the input to the plant.

(c) Is this system better at tracking references or rejecting disturbances? Explain your response briefly.

(d) Verify your results for parts (a) and (b) using MATLAB by plotting unit step and ramp responses for both tracking and disturbance rejection.

Solution:

(a) • P:

$$\frac{Y(s)}{R(s)} = \frac{k_p G(s)}{1 + k_p G(s)} = \frac{19}{5s^2 + 6s + 20}$$

$$E(s) = R(s) - Y(s) = \frac{5s^2+6s+1}{5s^2+6s+20} R(s)$$

$$e_{ss} = \lim_{s\to o} sE(s) = \lim_{s\to o} s\frac{5s^2+6s+1}{5s^2+6s+20} R(s) = \lim_{s\to o} s\frac{5s^2+6s+1}{5s^2+6s+20}\frac{1}{s^{k+1}}$$

$$e_{ss} = \lim_{s\to o} \frac{5s^2+6s+1}{5s^2+6s+20}\frac{1}{s^{k}}$$

$$k = 0, e_{ss} = \frac{1}{20} = \frac{1}{1+k_p}$$

therefore the system is type 0 and phase constant is $k_p = 19$

• PD:

$$\frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)} = \frac{19 + 4s}{5s^2 + 10s + 20}$$

$$E(s) = R(s) - Y(s) = \frac{5s^2+6s+1}{5s^2+10s+20} R(s)$$

$$e_{ss} = \lim_{s\to o} sE(s) = \lim_{s\to o} s\frac{5s^2+6s+1}{5s^2+10s+20} R(s) = \lim_{s\to o} s\frac{5s^2+6s+1}{5s^2+10s+20}\frac{1}{s^{k+1}}$$

$$e_{ss} = \lim_{s \to o} \frac{5s^2 + 6s + 1}{5s^2 + 10s + 20} \frac{1}{s^k}$$

$$k = 0, e_{ss} = \frac{1}{20} = \frac{1}{1 + k_p}$$

therefore the system is type 0 and phase constant is $k_p = 19$

- PID:

$$\frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)} = \frac{8s^2 + 38s + 19}{10s^3 + 20s^2 + 40s + 19}$$

$$E(s) = R(s) - Y(s) = \frac{10s^3 + 12s^2 + 2s}{10s^3 + 20s^2 + 40s + 19} R(s)$$

$$e_{ss} = \lim_{s \to o} sE(s) = \lim_{s \to o} s \frac{10s^3 + 12s^2 + 2s}{10s^3 + 20s^2 + 40s + 19} R(s) = \lim_{s \to o} s \frac{10s^3 + 12s^2 + 2s}{10s^3 + 20s^2 + 40s + 19} \frac{1}{s^{k+1}}$$

$$e_{ss} = \lim_{s \to o} \frac{10s^3 + 12s^2 + 2s}{10s^3 + 20s^2 + 40s + 19} \frac{1}{s^k}$$

$$k = 1, e_{ss} = \frac{2}{19} = \frac{1}{k_v}$$

therefore the system is type 1 and velocity constant is $k_v = 9.5$

(b)

- P:

$$\frac{Y(s)}{W(s)} = \frac{G(s)}{1 + k_p G(s)} = \frac{1}{5s^2 + 6s + 20}$$

$$\frac{E(s)}{W(s)} = -\frac{\theta(s)}{W(s)} = \frac{-1}{5s^2 + 6s + 20} = T_w(s) = s^0 T_{0,w}(s)$$

$$y_{ss} = \lim_{s \to 0} sT_w(s) \frac{1}{s^{k+1}} = \lim_{s \to 0} T_w(s) \frac{1}{s^k} = \lim_{s \to 0} T_{0,w}(s) \frac{s^0}{s^k}$$

$$n = 0, k = 0, y_{ss} = \frac{-1}{20}$$

therefore the system is type 0 and error constant is -20

- PD:

$$\frac{Y(s)}{W(s)} = \frac{G(s)}{1 + D(s)G(s)} = \frac{1}{5s^2 + 10s + 20}$$

$$\frac{E(s)}{W(s)} = -\frac{\theta(s)}{W(s)} = \frac{-1}{5s^2 + 10s + 20} = T_w(s) = s^0 T_{0,w}(s)$$

$$y_{ss} = \lim_{s \to 0} sT_w(s) \frac{1}{s^{k+1}} = \lim_{s \to 0} T_w(s) \frac{1}{s^k} = \lim_{s \to 0} T_{0,w}(s) \frac{s^0}{s^k}$$

$$n = 0, k = 0, y_{ss} = \frac{-1}{20}$$

therefore the system is type 0 and error constant is –20

- PID:

$$\frac{Y(s)}{W(s)} = \frac{G(s)}{1+D(s)G(s)} = \frac{2s}{10s^3+20s^2+40s+19}$$

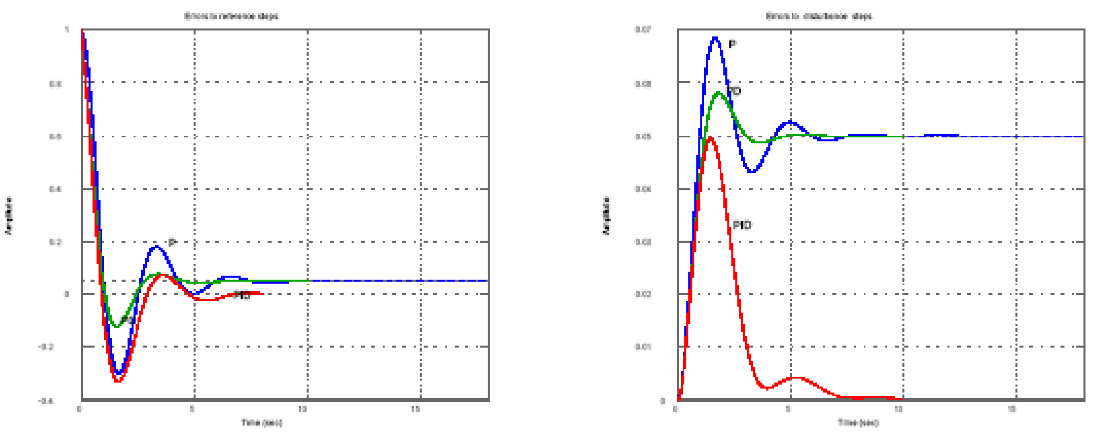
$$\frac{E(s)}{W(s)} = -\frac{\theta(s)}{W(s)} = \frac{-2s}{10s^3+20s^2+40s+19} = T_w(s) = s^1 T_{0,w}(s)$$

$$y_{ss} = \lim_{s\to 0} sT_w(s)\frac{1}{s^{k+1}} = \lim_{s\to 0} T_w(s)\frac{1}{s^k} = \lim_{s\to 0} T_{0,w}(s)\frac{s^1}{s^k}$$

$$n=1,\ k=1,\ y_{ss} = \frac{-2}{19}$$

therefore the system is type 1 and error constant is –9.5

(c) There is reduced oscillatory behavior brought on by addition of the derivative term, and increased oscillatory but lower error brought on by the integral term.
(d) See attached plots of the errors.



Problem 4.12(d)

21. Consider the second-order system

$$G(s) = \frac{1}{s^2+2\zeta s+1}.$$

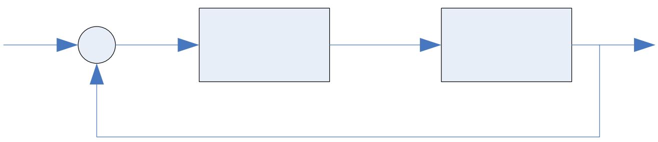
We would like to add a transfer function of the form $D(s) = K(s+a)/(s+b)$ in series with $G(s)$ in a unity-feedback structure.

(a) Ignoring stability for the moment, what are the constraints on K, a, and b so that system type 1?

(b) What are the constraints placed on K, a, and b so that the system is stable and type 1?

(c) What are the constraints on a and b so that the system is type 1 and remains stable for every positive value for K?

Solution:



(a)

$$D(s)=\frac{k(s+a)}{s+b},\quad G(s)=\frac{1}{s^2+2\xi s+1}$$

$$H(s)=\frac{D(s)G(s)}{1+D(s)G(s)}=\frac{\frac{k(s+a)}{s+b}\frac{1}{s^2+2\xi s+1}}{1+\frac{k(s+a)}{s+b}\frac{1}{s^2+2\xi s+1}}$$

$$=\frac{k(s+a)}{(s+b)(s^2+2\xi s+1)+k(s+a)}$$

$$=\frac{ks+ka}{s^3+(2\xi+b)s^2+(1+2b\xi+k)s+ka+b}$$

R(s) +

$$E(s)=R(s)-H(s)R(s)=\frac{1}{1+D(s)G(s)}R(s)$$

$$=\frac{s^3+(2\xi+b)s^2+(1+2b\xi)s+b}{s^3+(2\xi+b)s^2+(1+2b\xi+k)s+ka+b}R(s)$$

$$e_{ss}=\lim_{s\to 0}sE(s)=\lim_{s\to 0}s\frac{s^3+(2\xi+b)s^2+(1+2b\xi)s+b}{s^3+(2\xi+b)s^2+(1+2b\xi+k)s+ka+b}\frac{1}{s^{k+1}}$$

$$=\lim_{s\to 0}\frac{s^3+(2\xi+b)s^2+(1+2b\xi)s+b}{s^3+(2\xi+b)s^2+(1+2b\xi+k)s+ka+b}\frac{1}{s^k}$$

if system type = 1, then k = 1. e_{ss} is constant, so b=0 and ka+b$\neq$0

$\therefore\ b=0\ \ and\ \ ka\neq b$

(b) if system type = 1 $\Rightarrow$ b = 0 and $ka\neq b$

characteristic equation is:

$$s^3+(2\xi+b)s^2+(1+2b\xi+k)s+ka+b=0$$

b = 0, so it equals to:

$$s^3+2\xi s^2+(1+k)s+ka=0$$

s^3	1	$1+k$
s^2	2ξ	ka
s^1	$\frac{(1+k)2\xi-ka}{2\xi}$	0
s^0	ka	

$\therefore$ if system is stable, there must be:

ξ>0, ka>0, $(1+k)2\xi-ka$ >0

$$\begin{cases} \xi > 0 \\ ka > 0 \\ (1+k)2\xi > ka \end{cases}$$

(c) if system is stable and system type is 1, then there must be

$$\begin{cases} \xi > 0 \\ ka > 0 \\ (1+k)2\xi > ka \end{cases} \quad \text{if k>0 then} \quad \begin{cases} \xi > 0 \\ 0 < a < \dfrac{(1+k)2\xi}{k} \end{cases}$$

28. One possible representation of an automobile speed-control system with integral control is shown in Fig. 4.52.

(a) With a zero reference velocity input ($v_c = 0$), find the transfer function relating the output speed v to the wind disturbance w.

(b) What is the steady-state response of v if w is a unit ramp function?

(c) What type is this system in relation to reference inputs? What is the value of the corresponding error constant?

(d) What is the type and corresponding error constant of this system in relation to tracking the disturbance w?

Solution:

(a)

$$\frac{V(s)}{W(s)} = \frac{ms}{s^2 + mk_3 s + mk_1 k_2}$$

(b)

$$v_{ss} = \lim_{s \to 0} s \frac{V(s)}{W(s)} \frac{W_0}{s^2} = \frac{W_0}{k_1 k_2}$$

where $W(s) = \frac{W_0}{s^2}$.

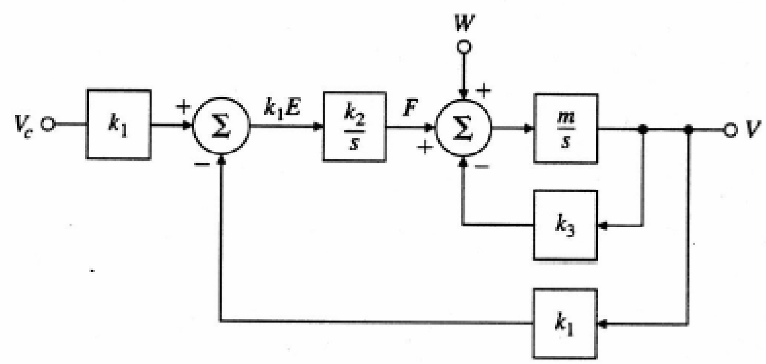


Figure 4.52: System using integral control

(c)

$$E = V_c - V = [1 - \frac{\frac{k_1 k_2 m}{s(s+mk_3)}}{1+\frac{k_1 k_2 m}{s(s+mk_3)}}]V_c = \frac{1}{1+\underbrace{\frac{mk_1k_2}{s(s+mk_3)}}_{G(s)}}V_c$$

$$e_{ss} = \lim_{s\to 0} \frac{sV_c}{1+G(s)}$$

$$K_p = \lim_{s\to 0} G(s) = \infty \Longrightarrow e_\infty(\text{step input}) = 0$$

$$K_v = \lim_{s\to 0} sG(s) = \frac{k_1k_2}{k_3} \Longrightarrow e_\infty(\text{ramp input}) = \frac{1}{K_v} = \frac{k_3}{k_1k_2}$$

System is Type 1.

error constant is $\frac{k_1k_2}{k_3}$

(d) $\frac{V(s)}{W(s)} = \frac{ms}{s^2+mk_3s+mk_1k_2}$

$$\frac{E(s)}{W(s)} = -\frac{V(s)}{W(s)} = \frac{-ms}{s^2+mk_3s+mk_1k_2} = T_w(s) = s^1T_{0,w}(s)$$

$$y_{ss} = \lim_{s\to 0} sT_w(s)\frac{1}{s^{k+1}} = \lim_{s\to 0} T_w(s)\frac{1}{s^k} = \lim_{s\to 0} T_{0,w}(s)\frac{s^1}{s^k}$$

$$n=1,\ k=1,\ y_{ss} = \frac{-1}{k_1k_2}$$

therefore the system is type 1 and error constant is $-k_1k_2$

34. Consider the system shown in Fig. 4.57.

(a) Find the transfer function from the reference input to the tracking error.

(b) For this system to respond to inputs of the form $r(t) = t^n 1(t)$ (where $n < q$) with zero steady-state error, what constraint is placed on the open-loop poles $p_1, p_2, \cdots, p_q$?

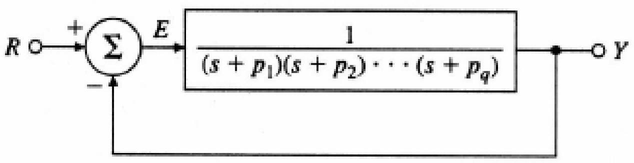


Figure 4.57: Control system for Problem 4.34

Solution:

(a) $$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = \frac{\prod_{i=1}^{q}(s+p_i)}{\prod_{i=1}^{q}(s+p_i)+1}$$

(b)

$$r(t) = t^n \quad \Rightarrow \quad R(s) = \frac{n!}{s^{n+1}}$$

$$e_{ss} = \lim_{s\to 0} s \frac{n!}{s^{n+1}} \frac{\prod_{i=1}^{q}(s+p_i)}{\prod_{i=1}^{q}(s+p_i)+1}$$

If e_{ss} is to be zero the system must have at least n+1 poles at the origin:

$$e_{ss} = \lim_{s\to 0} s \frac{n!}{s^{n+1}} \frac{s^{n+1}\prod_{i=1}^{q}(s+p_i)}{s^{n+1}\prod_{i=1}^{q}(s+p_i)+1} = 0$$