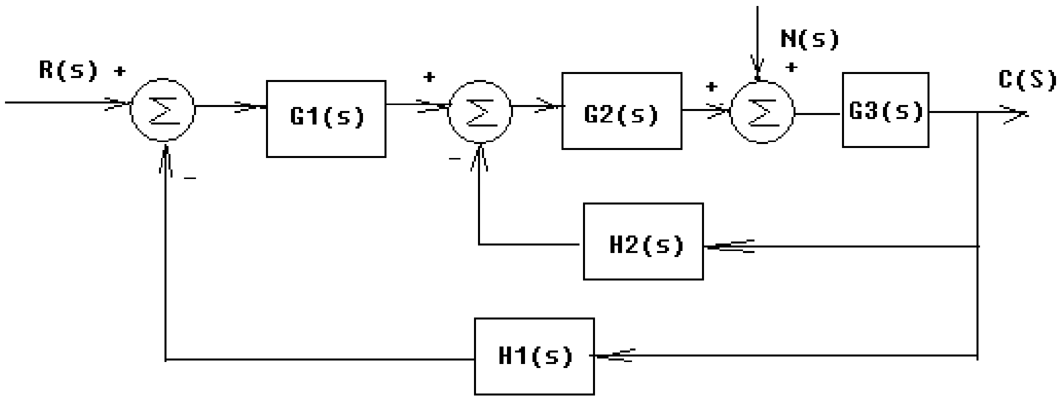
Part A

1. Solution:



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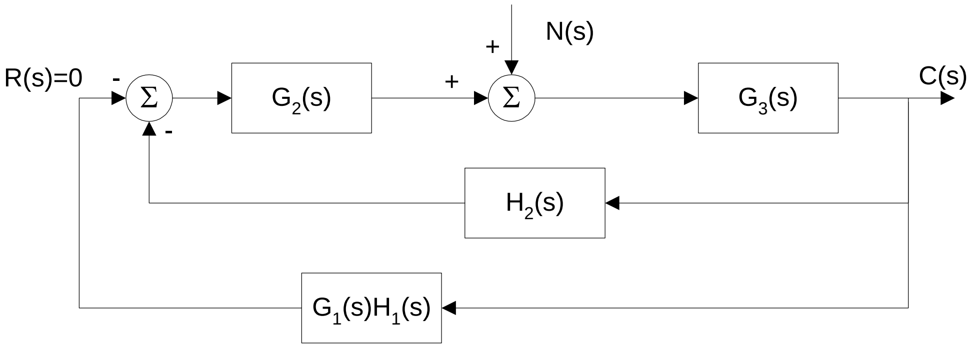

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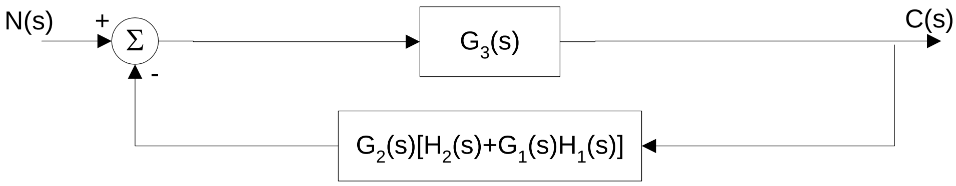


$$\therefore \frac{C(s)}{N(s)} = \frac{G_3(s)}{1+G_2(s)G_3(s)[H_2(s)+G_1(s)H_1(s)]}$$

2. Solution:

For system (a):

$$\frac{C(s)}{N(s)} = \frac{1}{1+KG(s)H(s)}$$

$$C(s) = \frac{1}{1+KG(s)H(s)} N(s)$$ (G(s) and H(s) are fixed)

From the above equation, we could know when K is larger, the influence of disturbance N(s) is smaller, which means the sensitivity of C(s)/N(s) is smaller.

For System (b):

$$\frac{C(s)}{N(s)} = \frac{-KG(s)H(s)}{1-(-KG(s)H(s))} = \frac{-KG(s)H(s)}{1+KG(s)H(s))}$$

$$C(s) = -\frac{G(s)H(s)}{\frac{1}{K}+G(s)H(s))}N(s)$$

H(s) and G(s) are fixed, when K is larger, $\frac{C(s)}{N(s)}$ is larger. So the result for (a) is not correct for (b).

4. The DC-motor speed control in Fig. 4.38 is described by the differential equation

$$\dot{y} + 60y = 600v_a - 1500w,$$

where y is the motor speed, v_a is the armature voltage, and w is the load torque. Assume the armature voltage is computed using the PI control law

$$v_a = (k_p e + k_I \int_0^t e dt)$$

where $e = r - y$.

(a) Compute the transfer function from W to Y as a function of k_p and k_I.

(b) Compute values for k_p and k_I so that the characteristic equation of the closed-loop system will have roots at $-60 \pm 60j$.

Solution:

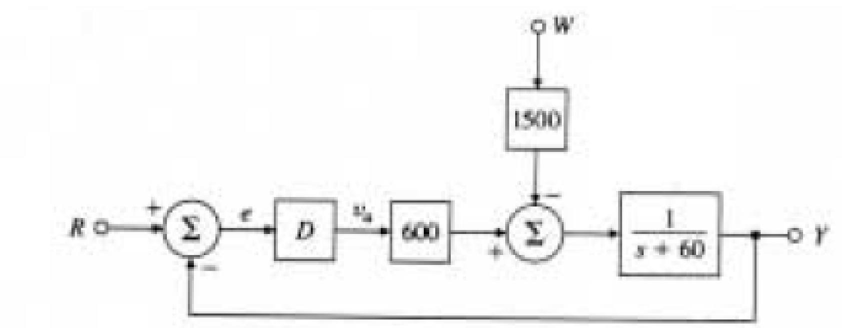


Figure 4.38: Unity feedback system with prefilter for Problem 4.4

(a) Transfer function: Set $R = 0$, then $E = -Y$

$$\begin{aligned}(s+60)Y(s) &= -600[k_p Y(s) + \frac{k_I}{s}Y(s)] - 1500W(s) \\ \frac{Y(s)}{W(s)} &= \frac{-1500s}{s^2 + 60(1+10k_p)s + 600k_I}\end{aligned}$$

(b) For roots at $-60 \pm j60$: comparing to the standard form:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \implies s = -\zeta\omega_n \pm j\omega_n \sqrt{1-\zeta^2}$$

$$\omega_n = 60\sqrt{2}\ , \quad \zeta = 0.707$$

$$600k_I = (60\sqrt{2})^2 \implies k_I = 12$$

$$60(1+10k_p) = 2 \times 0.707 \times 60\sqrt{2} \implies k_p = 0.1$$

5. Consider the system shown in Fig. 4.39, which consists of a prefilter and a unity feedback system.

   (a) Determine the transfer function from R to Y.

   (b) Determine the steady-state error due to a step input.

   (c) Discuss the effect of different values of (K_r, a) on the system's response.

   (d) For each of the following three cases,

   $$(1)\ A = 1,\ \tau = 1, \quad (2)\ A = 10,\ \tau = 1, \quad (3)\ A = 1,\ \tau = 2,$$

   use MATLAB to find values for K_r and a so that (if possible)

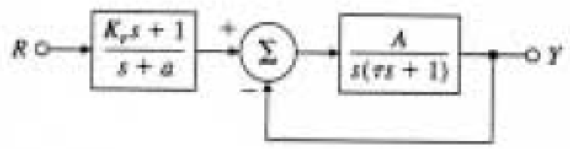


Figure 4.39: Block diagrams for Problem 4.5

   i. the rise time is less than 1.5 sec.,
   ii. the overshoot is less than 20%, ,
   iii. the settling time is less than 10 sec. and
   iv. the steady-state error is less than 5%.

In cases where the specifications are easily met, try to make the rise time as small as possible. If the specifications cannot be met, find the design to meet as many of the specifications as possible, in the order given.

Solution:

(a)

$$Y = \frac{A}{s(\tau s+1)}E = \frac{A}{s(\tau s+1)}(\frac{K_r s+1}{s+a}R - Y)$$

or

$$(1 + \frac{A}{s(\tau s+1)})Y = \frac{A(K_r s+1)}{s(s+a)(\tau s+1)}R$$

$$Y = \frac{A(K_r s+1)}{s(s+a)(\tau s+1)}\frac{s(\tau s+1)}{s(\tau s+1)+A}R$$

$$Y = \frac{A(K_r s+1)}{(s+a)\left[s(\tau s+1)+A\right]}R = \frac{AK_r s+A}{\tau s^3+(1+a\tau)s^2+(a+A)s+Aa}R$$

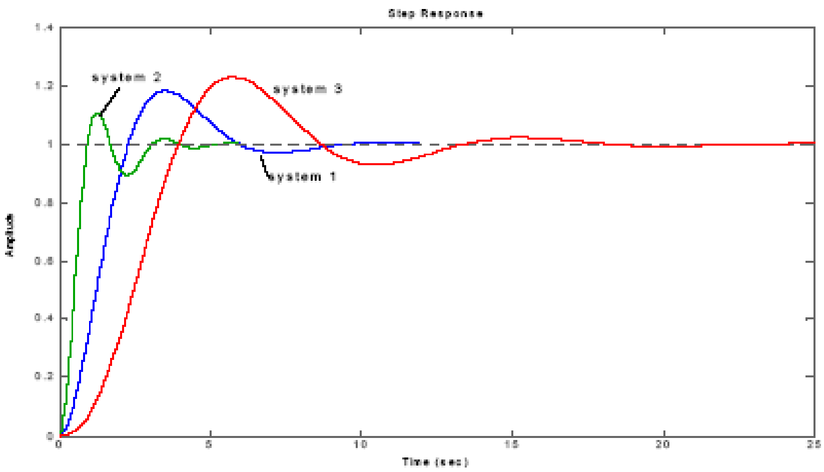
(b)

$$y(\infty) = \lim_{s\to 0} s\frac{1}{s}\frac{AK_r s+A}{\tau s^3+(1+a\tau)s^2+(a+A)s+Aa} = \frac{1}{a}$$

$$e_{ss,step} = \frac{1}{a} - 1 = \frac{1-a}{a}$$

(c) K_r determines the prefilter zero location and can have a significant effect on overshoot. The prefilter pole location a will affect the speed of the transient response and the size of the steady-state error to a step.

- $A = 1, \tau = 1 : K_r = 1.1, a = 1$ yields $t_r = 1.51$ sec, $M_p = 18\%$, $t_s = 9$ sec.
- $A = 10, \tau = 1 : K_r = 0.555,\ a = 1$ yields response well within specs, with min. rise time of 0.53 sec.
- $A = 1, \tau = 2$: specs cannot be met. $K_r = 0.1, a = 1$ results in 23% overshoot and $t_s = 17$ sec and $t_r = 2.4$ sec. See the following step responses.



6. A unity feedback control system has the open-loop transfer function

$$G(s) = \frac{A}{s(s+a)}.$$

(a) Compute the sensitivity of the closed-loop transfer function to changes in the parameter A.

(b) Compute the sensitivity of the closed-loop transfer function to changes in the parameter a.

(c) If the unity gain in the feedback changes to a value of $\beta \neq 1$, compute the sensitivity of the closed-loop transfer function with respect to β.

(d) Assuming $A = 1$ and $a = 1$, plot the magnitude of each of the above sensitivity functions for $s = j\omega$ using **semilogy** command in MATLAB. Comment on the relative effect of parameter variations in A, a, , and β at different frequencies ω, paying particular attention to DC (when $\omega = 0$).

Solution:

(a)

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{A}{s(s+a)}}{1+\frac{A}{s(s+a)}} = \frac{A}{s^2+as+A}$$

$$\frac{dT}{dA} = \frac{(s^2+as+A)-A}{(s^2+as+A)^2}$$

$$\mathcal{S}_A^T = \frac{A}{T}\frac{dT}{dA} = \frac{A(s^2+as+A)}{A}\frac{s^2+as}{(s^2+as+A)^2} = \frac{s(s+a)}{s(s+a)+A}$$

(b)

$$\frac{dT}{da} = \frac{-sA}{(s^2+as+A)^2}$$

$$\frac{a}{T}\frac{dT}{da} = \frac{a(s^2+as+A)}{A}\frac{-sA}{(s^2+as+A)^2}$$

$$\mathcal{S}_a^T = \frac{-as}{s(s+a)+A}$$

(c) In this case,

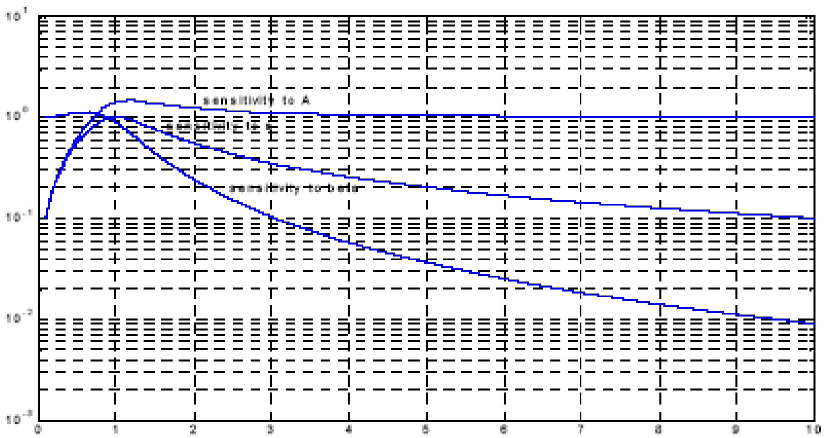
$$T(s) = \frac{G(s)}{1 + \beta G(s)}$$

$$\frac{dT}{d\beta} = \frac{-G(s)^2}{(1 + \beta G(s))^2}$$

$$\frac{\beta}{T}\frac{dT}{d\beta} = \frac{\beta(1+\beta G)}{G}\frac{-G^2}{(1+\beta G)^2} = \frac{-\beta G}{1+\beta G}$$

$$\mathcal{S}_\beta^T = \frac{\dfrac{-\beta A}{s(s+A)}}{1 + \dfrac{\beta A}{s(s+a)}} = \frac{-\beta A}{s(s+a) + \beta A}$$

- Transfer function is most sensitive to variations in a and A near $\omega = 1$ rad/sec (due to the fact that $a = 1$).
- Steady-state response is not affected by variations in A and a ($\mathcal{S}_A^T(0)$ and $\mathcal{S}_a^T(0)$ are both zeros).
- Steady-state response is heavily dependent on β since $|\mathcal{S}_\beta^T(0)| = 1$. See attached plots



Problem 4.6 (d)