24. For the unity feedback system shown in Fig. 3.59, specify the gain K of the proportional controller so that the output $y(t)$ has an overshoot of no more than 10% in response to a unit step.

Solution:

$$\begin{aligned} \frac{Y(s)}{R(s)} &= \frac{K}{s^2+2s+K} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \\ \omega_n &= \sqrt{K} \\ \zeta &= \frac{2}{2\omega_n} = \frac{1}{\sqrt{K}} \qquad (1) \end{aligned}$$

In order to have an overshoot of no more than 10%:

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \leq 0.10$$

Solving for ζ :

Figure 3.60: Unity feedback system for Problem 3.25

$$\zeta = \sqrt{\frac{(\ln M_p)^2}{\pi^2+(\ln M_p)^2}} \geq 0.591$$

Using (1) and the solution for ζ:

$$K = \frac{1}{\zeta^2} \leq 2.86$$

$$\therefore 0 < K \leq 2.86$$

25. For the unity feedback system shown in Fig. 3.60, specify the gain and pole location of the compensator so that the overall closed-loop response to a unit-step input has an overshoot of no more than 25%, and a 1% settling time of no more than 0.1 sec. Verify your design using MATLAB.

Solution:

$$\frac{Y(s)}{R(s)} = \frac{100K}{s^2 + (25+a)s + 25a + 100K} = \frac{100K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Using the given information:

$$\begin{aligned} R(s) &= \frac{1}{s} \qquad \text{unit step} \\ M_p &\le 25\% \\ t_{1\%} &\le 0.1\,\text{sec} \end{aligned}$$

Solve for ζ :

$$\begin{aligned} M_p &= e^{-\pi\zeta/\sqrt{1-\zeta^2}} \\ \zeta &= \sqrt{\frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}} \ge 0.4037 \end{aligned}$$

Solve for ω_n:

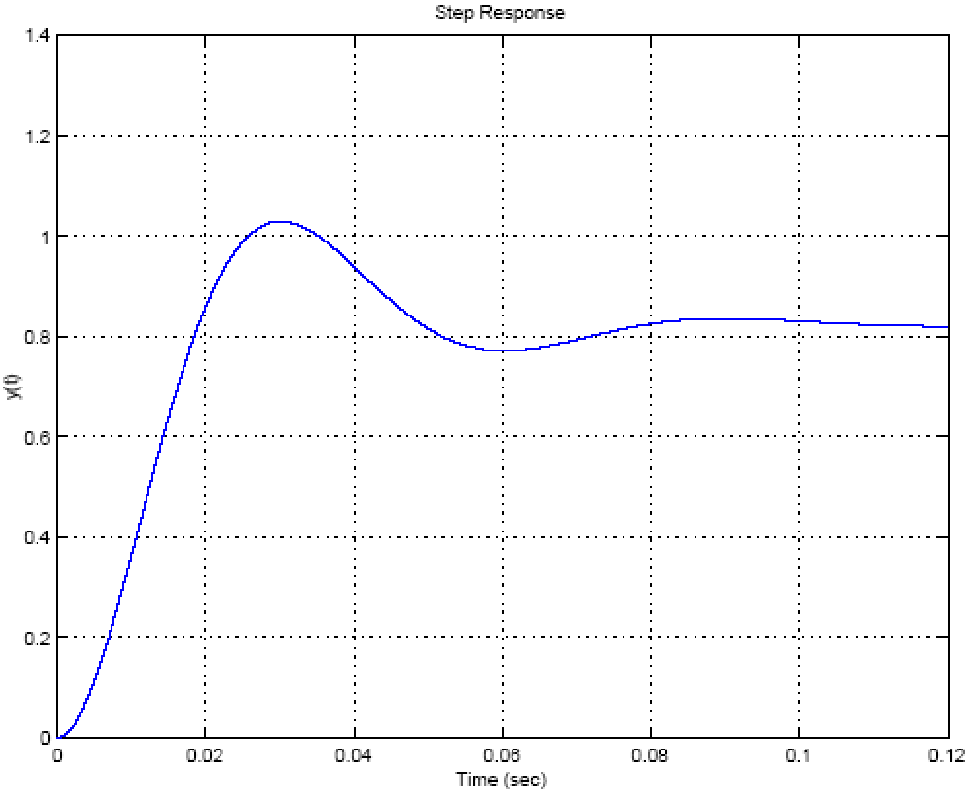
$$e^{-\zeta\omega_n t_s} = 0.01 \qquad \text{For a 1\% settling time}$$

$$\begin{aligned} t_s \le \frac{4.605}{\zeta\omega_n} &= 0.1 \\ \Longrightarrow \omega_n &\approx 114.07 \end{aligned}$$

Now find a and K :

$$\begin{aligned} 2\zeta\omega_n &= (25+a) \\ a = 2\zeta\omega_n - 25 &= 92.10 - 25 = 67.10 \\ \omega_n^2 &= (25a + 100K) \\ K = \frac{\omega_n^2 - 25a}{100} &\approx 113.34 \end{aligned}$$

The step response of the system using MATLAB is shown below.



Step Response for Problem 3.25

40. Use Routh's stability criterion to determine how many roots with positive real parts the following equations have.

    (a) $s^4 + 8s^3 + 32s^2 + 80s + 100 = 0.$

    (b) $s^5 + 10s^4 + 30s^3 + 80s^2 + 344s + 480 = 0.$

    (c) $s^4 + 2s^3 + 7s^2 - 2s + 8 = 0.$

    (d) $s^3 + s^2 + 20s + 78 = 0.$

    (e) $s^4 + 6s^2 + 25 = 0.$

Solution:

(a)

$$s^4 + 8s^3 + 32s^2 + 80s + 100 = 0$$

$$\begin{array}{llll}
s^4 & : & 1 & 32 & 100 \\
s^3 & : & 8 & 80 \\
s^2 & : & 22 & 100 \\
s^1 & : & 80 - \dfrac{800}{22} = 43.6 \\
s^0 & : & 100
\end{array}$$

$\Longrightarrow$ No roots not in the LHP

(c)

$$s^4 + 2s^3 + 7s^2 - 2s + 8 = 0$$

There are roots in the RHP (not all coefficients are >0).

(d) The Routh array is,

$$\begin{array}{llrl}
s^3 & : & 1 & 20 \\
s^2 & : & 1 & 78 \\
s^1 & : & -58 \\
s^0 & : & 78
\end{array}$$

There are two sign changes in the first column of the Routh array. Therefore, there are two roots not in the LHP.

(e)

$$a(s) = s^4 + 6s^2 + 25 = 0$$

Two coefficients are missing so there are roots outside the LHP.

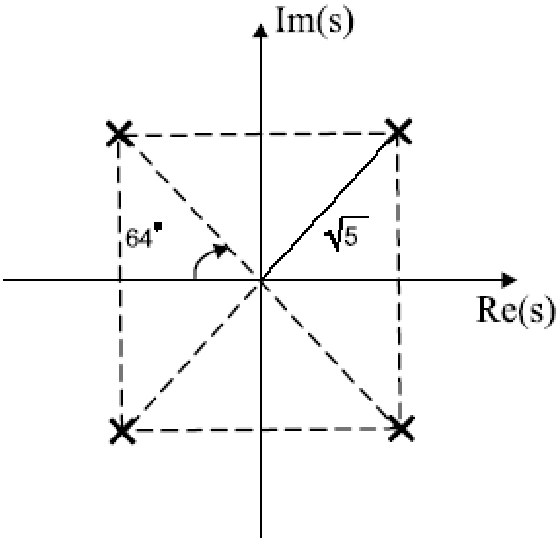
Create a new row by $\frac{da(s)}{ds}$

$$\begin{array}{lllll}
s^4 & : & 1 & 6 & 25 \\
s^3 & : & 4 & 12 & \longleftarrow \text{new row} \\
s^2 & : & 3 & 25 \\
s^1 & : & 12 - \dfrac{100}{3} = -21.3 \\
s^0 & : & 25
\end{array}$$

$\Longrightarrow$ 2 roots not in the LHP

check:

$$\begin{aligned} a(s) &= 0 \Longrightarrow s^2 = -3 \pm 4j = 5e^{j(\pi \pm 0.92)} \\ s &= \sqrt{5}e^{j\left(\frac{\pi}{2} \pm 0.46\right) + n\pi j} \qquad n = 0, 1 \end{aligned}$$



Problem 3.40: s-plane pole locations

43. Consider the system shown in Fig. 3.69.

   (a) Compute the closed-loop characteristic equation.

   (b) For what values of (T, A) is the system stable? **Hint**: An approximate answer may be found using

$$e^{-Ts} \cong 1 - Ts$$

or

$$e^{-Ts} \cong \frac{1 - \frac{T}{2}s}{1 + \frac{T}{2}s}$$

for the pure delay. As an alternative, you could use the computer MATLAB (Simulink) to simulate the system or to find the roots of the system's characteristic equation for various values of T and A.

Solution:

(a) The characteristic equation is,

$$s(s+1) + Ae^{-Ts} = 0$$

(b) Using $e^{-Ts} \cong 1 - Ts$, the characteristic equation is,

$$s^2 + (1 - TA)s + A = 0$$

The Routh's array is,

$$\begin{array}{lcll} s^2 & : & 1 & A \\ s^1 & : & 1 - TA & 0 \\ s^0 & : & A & \end{array}$$

For stability we must have $A > 0$ and $TA < 1$.

Using $e^{-Ts} \cong \frac{(1-\frac{T}{2}s)}{(1+\frac{T}{2}s)}$, the characteristic equation is,

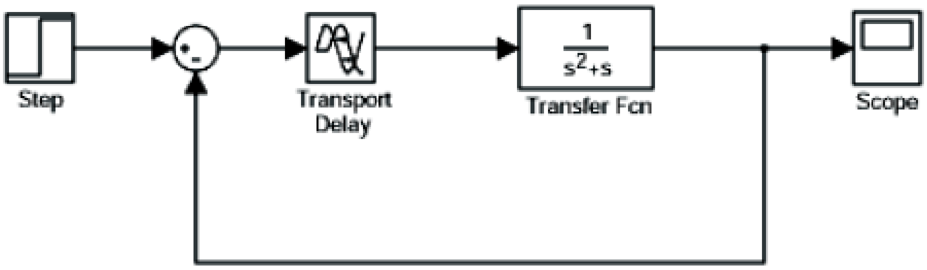
$$s^3 + (1 + \frac{2}{T})s^2 + (\frac{2}{T} - A)s + \frac{2}{T}A = 0$$

The Routh's array is,

$$\begin{array}{lcll} s^3 & : & 1 & (\frac{2}{T} - A) \\ s^2 & : & (1 + \frac{2}{T}) & \frac{2A}{T} \\ s^1 & : & \frac{(1 + \frac{2}{T})(\frac{2}{T} - A) - \frac{2A}{T}}{(1 + \frac{2}{T})} & 0 \\ s^0 & : & \frac{2A}{T} & \end{array}$$

For stability we must have all the coefficients in the first column be positive.

The following Simulink diagram simulates the system.



Problem 3.43: Simulink simulation diagram

44. Modify the Routh criterion so that it applies to the case where all the poles are to be to the left of $-\alpha$ when $\alpha > 0$. Apply the modified test to the polynomial

$$s^3 + (6+K)s^2 + (5+6K)s + 5K = 0,$$

finding those values of K for which all poles have a real part less than -1.

Solution:

Let $p = s + \alpha$ and substitute $s = p - \alpha$ to obtain a polynomial in terms of p. Apply the standard Routh test to the polynomial in p.

For the example $p = s + 1$ or $s = p - 1$. Substitute this in the polynomial,

$$(p-1)^3 + (6+K)(p-1)^2 + (5+6K)(p-1) + 5K = 0$$

or

$$p^3 + (3+K)p^2 + (4K-4)p \qquad = 0.$$

The Routh's array is,

$$\begin{array}{lclll}
p^3 & : & 1 & 4K-4 & \\
p^2 & : & 3+K & 0 & \\
p^1 & : & 4K\text{-}4 & & 0 \\
p^0 & : & 0 & &
\end{array}$$

Not all the first rows are greater than 0. So for any K, it's impossible that all the poles are to be the left of -1.