Problem 1.

$$H(s) = \frac{G(s)}{1+G(s)}$$

$$E(s) = R(s) - Y(s) = R(s)\,(1 - H(s)) = \frac{1}{1+G(s)} R(s) = \frac{1}{1 + \frac{2(s+3)}{s^n(s+1)(s+4)}} R(s)$$

$$= \frac{s^n(s+1)(s+4)}{s^n(s+1)(s+4) + 2(s+3)} R(s)$$

a) $e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{s^n(s+1)(s+4)}{s^n(s+1)(s+4) + 2(s+3)}$

if $e_{ss} = 0$ then $n \geq 1$

C.F. $s^n(s+1)(s+4) + 2(s+3) = 0$

when $n \geq 3$, system is not stable.

$n=1$, $s^3 + 5s^2 + 6s + 6 = 0$

s^3	1	6
s^2	5	6
s^1	$\frac{24}{5}$	0
s^0	6	

system is stable for $n=1$

$n=2$, $s^4 + 5s^3 + 4s^2 + 2s + 6 = 0$

s^4	1	4	6
s^3	5	2	
s^2	$\frac{18}{5}$	6	
s^1	$-\frac{57}{9}$		
s^0	6		

$\therefore$ system is not stable

$\therefore$ $n = 1$

b) $n=2$, system is not stable.

Problem 2.

$$G(s) = \frac{4s+1}{s^2(2s+5)}$$

$$H(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{4s+1}{s^2(2s+5)}}{1+\frac{4s+1}{s^2(2s+5)}} = \frac{4s+1}{2s^3+5s^2+4s+1}$$

$$= \frac{4s+1}{2s^3+5s^2+2s+2s+1}$$

$$= \frac{4s+1}{s(s+2)(2s+1)+(2s+1)}$$

$$= \frac{4s+1}{(2s+1)(s^2+2s+1)}$$

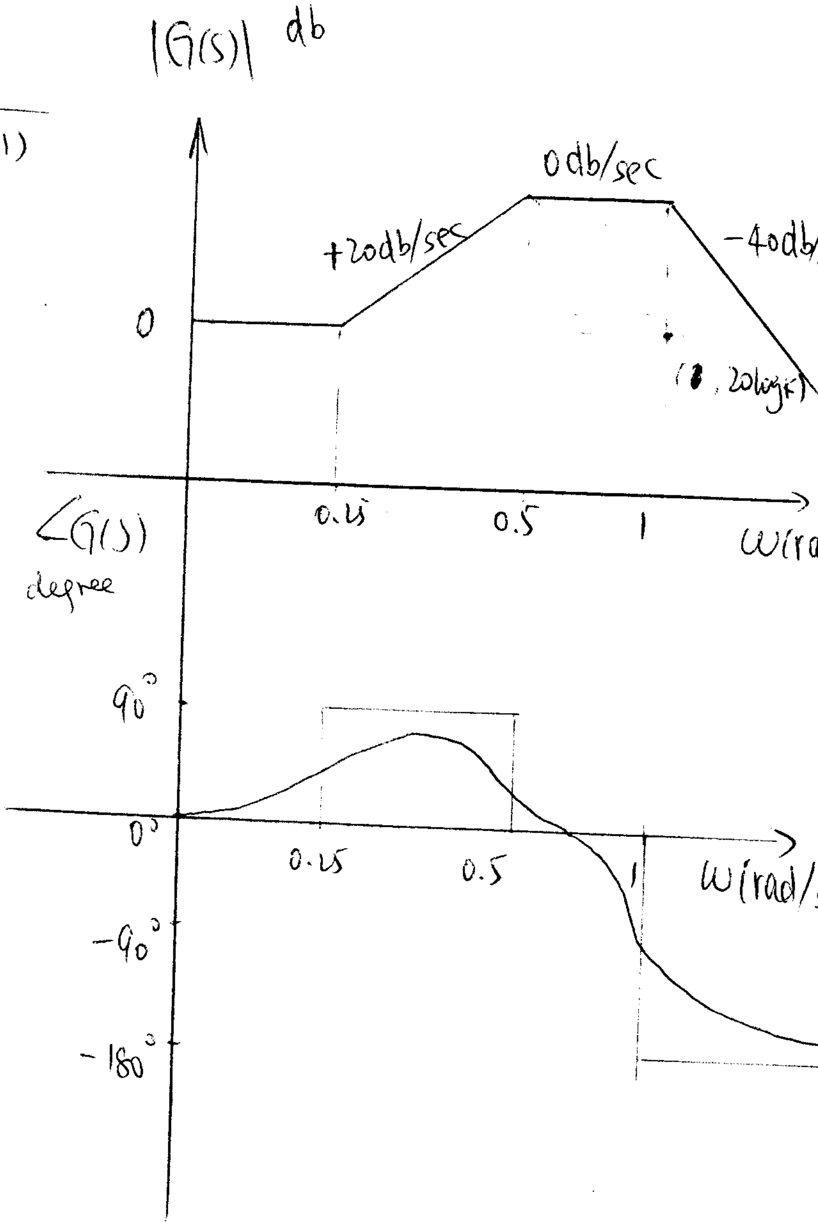
$$= \frac{4s+1}{(2s+1)(s+1)^2}$$

$$= \frac{\frac{s}{0.25}+1}{(\frac{s}{0.5}+1)(s+1)^2}$$

break point frequencies.

$\omega = 0.25, \quad \omega = 0.5, \quad \omega = 1$

$20 \log k = 0$



Problem 3

$Z = N + P$

a) $P = 2$

$N = Z - P = 1$

b) $P = 2$

$N = 1$

$Z = N + P = 3$

c) $P = Z - N$

$N = -2$

$Z = 0$

$P = 2$

Problem 4.

from Nyquist plot, $N = 2$, $P = 0$, $Z = N + P = 2$

$\therefore$ the closed loop unity feedback system is not stable

$$G(s) = \frac{k}{(s+1)(s+2)(s+6)} = \frac{k}{s^3 + 9s^2 + 20s + 12}$$

$s = j\omega$

$$G(j\omega) = \frac{k}{(j\omega)^3 + 9(j\omega)^2 + 20(j\omega) + 12}$$

$$= \frac{k}{12 - 9\omega^2 + (20\omega - \omega^3)j}$$

for the point on the real axis.

$$20\omega - \omega^3 = 0 \quad \Rightarrow \quad \omega = 0, \quad \omega^2 = 20$$

$\omega = 0$. $G(j\omega) = \frac{k}{12}$ on the right side of real axis

$\omega^2 = 20$ $\quad G(j\omega) = \frac{k}{12 - 180} = -\frac{k}{168}$

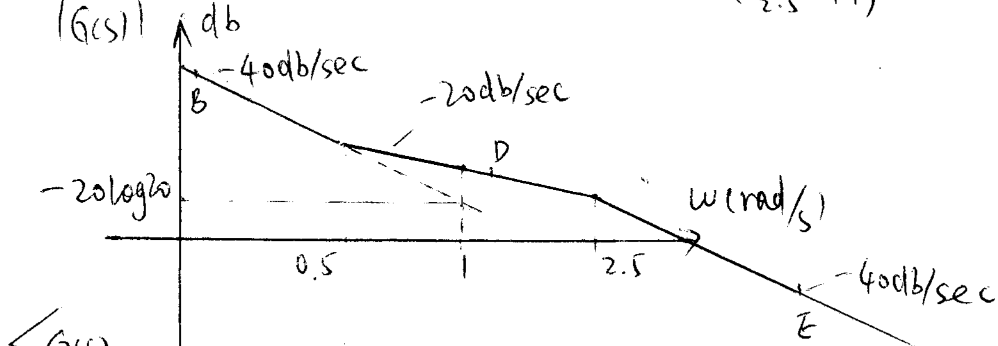
$\therefore$ when $-\frac{k}{168} > -1$

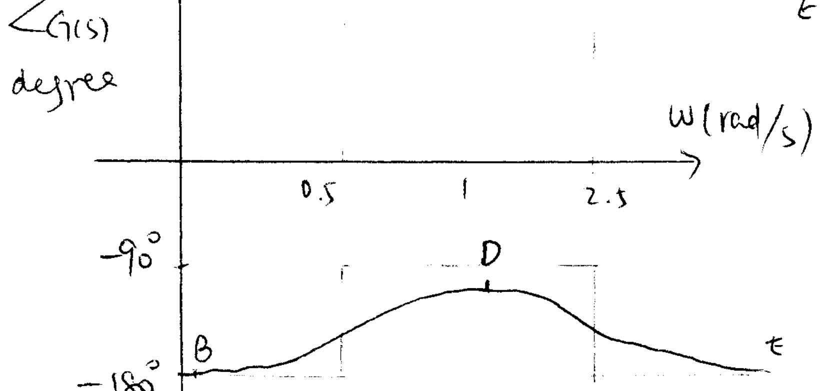
$\Rightarrow$ $k < 168$ then the system is stable

$\therefore$ $0 < k < 168$

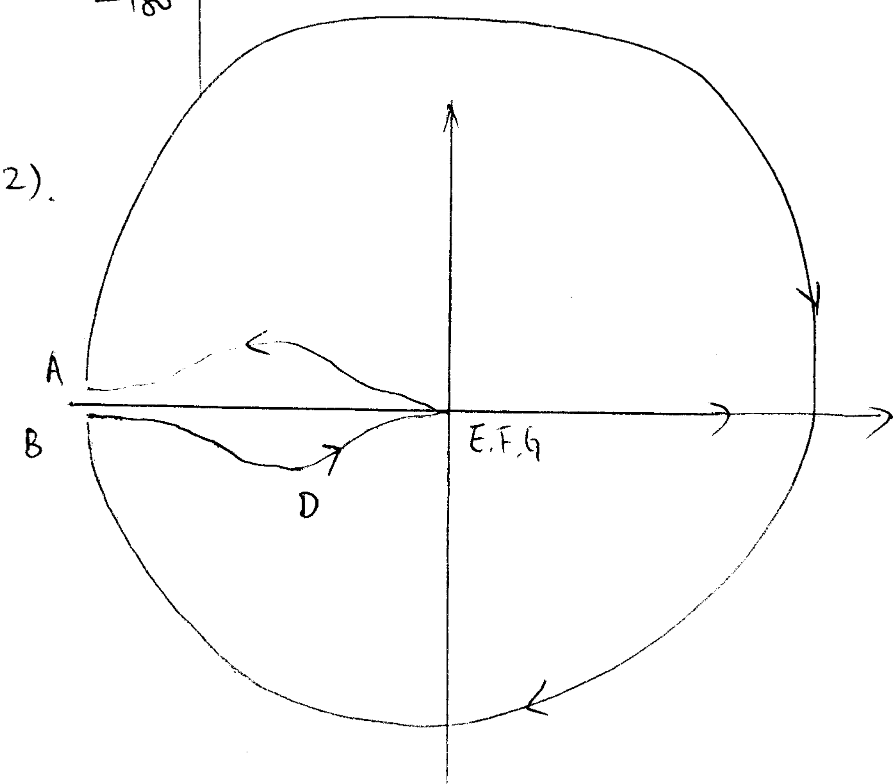
Problem 5.

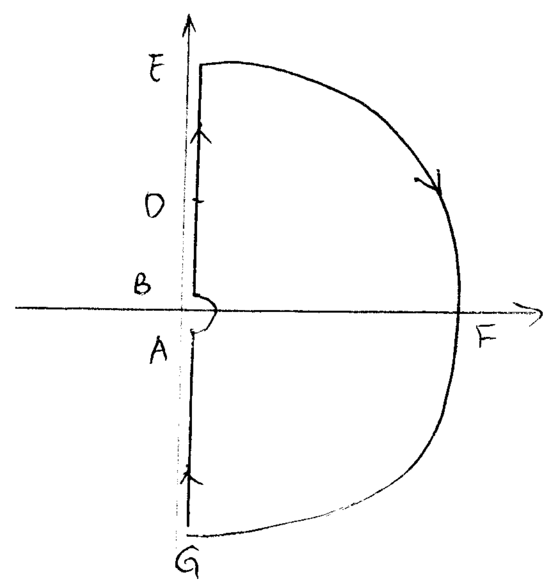
1) $$G(s)=\frac{s+0.5}{2s^2(2s+5)}=\frac{0.5\left(\frac{s}{0.5}+1\right)}{2s^2\times 5\left(\frac{s}{2.5}+1\right)}=\frac{\frac{s}{0.5}+1}{20s^2\left(\frac{s}{2.5}+1\right)}$$





2).





$\omega=0 \quad |G(s)|=\infty \quad \angle G(s)=-180^\circ$

$A\to B \quad s=\epsilon e^{j\theta} \quad G(s)=\dfrac{0.5}{2\epsilon^2 e^{2j\theta}\cdot 5}$

$A\to B: \quad \theta: -90^\circ - 90^\circ$

$\angle G(s): \quad 180^\circ \to -180^\circ$

$\omega=\infty \quad |G(s)|=0$

from nyquist plot, $K>0$ system is stable for all

$$1 + kG(s) = 0$$

$$1 + \frac{k(s+0.5)}{2s^2(5+2s)} = 0$$

$$2s^2(2s+5) + k(s+0.5) = 0$$

$$4s^3 + 10s^2 + ks + 0.5k = 0$$

$$8s^3 + 20s^2 + 2ks + k = 0$$

$$\begin{array}{llll}
s^3 & 8 & & 2k \\
s^2 & 20 & & k \\
s^1 & \dfrac{40k - 8k}{20} & = 1.6k & 0 \\
s^0 & k & &
\end{array}$$

if system is stable, the first column should be posit

$\therefore \quad k > 0$

$k > 0$, system is stable.

Problem 6

$$L(s) = \frac{k(s^2-3s+2)}{s^2+2s+2}$$

$a(s) = s^2+2s+2 = (s+1)^2 - j^2 \Rightarrow P_1 = -1+j, \; P_2 = -1-j$

$b(s) = s^2-3s+2 = (s-1)(s-2) \Rightarrow z_1 = 1, \; z_2 = 2.$ m

1). there are 2 branches start at the poles and

2 end on the zeros.

2). the real axis segments is between $+1 < s < 2$

3). $\alpha = \frac{\Sigma P_i - \Sigma z_i}{n-m}$

$n-m=0$, there 's no asymptote.

4). RULE 5.

$$1 + \frac{k(s^2-3s+2)}{s^2+2s+2} = 0$$

$s^2+2s+2 + k(s^2 - 3ks + 2k = 0$

$(1+k)s^2 + (2-3k)s + (2+2k) = 0$ (1)

s^2	$1+k$	$2+2k$
s^1	$2-3k$	0
s^0	$2+2k$	

$2-3k=0 \Rightarrow k=\frac{2}{3}$. Substitute into (1)

$$\frac{5}{3}s^2+\frac{10}{3}=0$$

$$s^2+2=0$$

$s=j\omega \Rightarrow (j\omega)^2+2=0 \Rightarrow \omega^2=2$

$\therefore$ for $k=\frac{2}{3}$, ② $\omega=\pm\sqrt{2}$, ② the roots are on the imaginary axis.

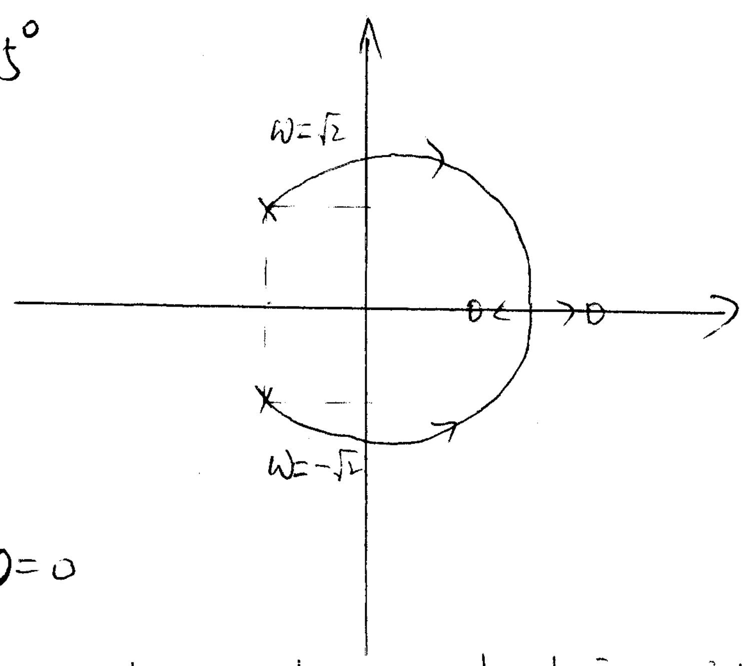
RULE 4.

C).4 Departure angle of $s=-1+j$

$$q\phi_{dep}=\Sigma\psi_i-\Sigma\phi_i-180^\circ+360^\circ(l-1)$$

$$=\arctan\frac{1}{-1-1}+\arctan\frac{1}{-1-2}-90^\circ-180^\circ+360^\circ(l-1)$$

$$=153.4^\circ+161.6^\circ-90^\circ-180^\circ=45^\circ$$



departure angle for $s=-1-j$ is

$$q\phi_{dep}=-45^\circ$$

RULE 6. break-in point

$b\frac{da}{ds}-a\frac{db}{ds}=0 \Rightarrow -5s^2+10=0$

$\Rightarrow s=\pm\sqrt{\frac{10}{5}}$ $s=\sqrt{\frac{10}{5}}=\sqrt{2}$ is on the root locus, $\therefore$ break-in point