

# 1. RC step response

$R = 2k\Omega$  Amplitude of  $V_{in} = 2V$

$C = 0.1\mu F$

1)  $V_{in} = 2u(t)$

$V_{out} = 2(1 - e^{-\frac{1}{RC}t})u(t) = 2(1 - e^{-5000t})u(t)$

5 points

2) See attached figure.

5 points

3)  $t(10\% V_{out}) = 2 \times 10^{-6} s = 2 \mu s = 2.021 ms$

$t(50\% V_{out}) = 0.139 \times 10^{-3} s = 139 \mu s$

10 points

$t(90\% V_{out}) = 0.461 \times 10^{-3} s = 461 \mu s$

$t_r = t(90\% V_{out}) - t(10\% V_{out}) = 0.44 ms$

$t_{fall} = 0.44 ms$

$t_{PLH} = 0.139 ms$       $t_{PHL} = 0.139 ms$

5 points

4)

# 2) LRC step response

$R = 2k\Omega$     $C = 0.1\mu F$     $L = 0.8 H$

$V_{in} = 2u(t)$

1)  $V_{in} = L \frac{di}{dt} + R \cdot i + V_{out}$   
 $i = C \frac{dV_{out}}{dt}$

10 points

$\Rightarrow \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{LCs^2 + RCs + 1} = \frac{1/LC}{s^2 + 2 \cdot \frac{R}{2L}s + (\frac{1}{LC})^2} = \frac{1.25 \times 10^7}{s^2 + 2 \times 1250s + (3535.5)^2}$

$\Rightarrow \omega_n = 3535.5$     $\xi = 0.3535$     $\omega_d = 3307$     $\sigma = \xi \cdot \omega_n = 1250$

$V_{out} = 2 \left( 1 - e^{-1250t} (\cos 3307t + 0.378 \sin 3307t) \right)$

5 points

2).

3)  $T_d$  (calculated) =  $T_d = \frac{1}{\omega_d} = \frac{2\pi}{\omega_d} = 1.9 ms$

4)  $f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$     $\xi = \frac{R}{2L \cdot \omega_n} = \frac{R\sqrt{C}}{2\sqrt{L}}$

5)  $f_n = 563 Hz$     $\xi = 0.3535$

6)  $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$     $0 \leq \xi < 1 \Rightarrow \xi = \sin\left(\tan^{-1} \frac{\ln M_p}{\pi}\right)$     $M_p = 0.60$  (calculated)

7)  $R \uparrow \rightarrow \xi \uparrow$  and  $\omega_d \downarrow \rightarrow M_p \downarrow$  but  $t_s \uparrow$ . If  $R$  is too big, no overshoot

8)  $C \uparrow \rightarrow \omega_n \downarrow, \zeta \uparrow, \omega_d = \sqrt{\frac{1}{LC}} \sqrt{1 - \frac{R^2 C}{4L}} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \downarrow$   
 $\rightarrow M_p \downarrow$  and  $t_d \uparrow$ . If  $C$  is too big, no overshoot.