Problem 1.

Answer (4).

$\because$ Transfer function $H(S) = \dfrac{\dfrac{1}{T_1 S+1} \cdot \dfrac{1}{T_2 S+1} \cdot \dfrac{1}{T_3 S+1}}{1+ \dfrac{1}{T_1 S+1} \cdot \dfrac{1}{T_2 S+1} \cdot \dfrac{1}{T_3 S+1}}$

$$= \frac{1}{(T_1 S+1)(T_2 S+1)(T_3 S+1) +1}$$

Characteristic function:

$$1+ (T_1 S+1)(T_2 S+1)(T_3 S+1) = 0$$

$$T_1 T_2 T_3 S^3 + (T_1 T_2 + T_1 T_3 + T_2 T_3) S^2 + (T_1 + T_2 + T_3) S + 2 = 0$$

$T_1, T_2, T_3 \neq 0$

$$\therefore \quad S^3 + \left(\frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{T_3}\right) S^2 + \left(\frac{1}{T_1 T_2} + \frac{1}{T_2 T_3} + \frac{1}{T_1 T_3}\right) S + \frac{2}{T_1 T_2 T_3} = 0$$

Routh's table:

S^3	1	$\frac{1}{T_1 T_2} + \frac{1}{T_2 T_3} + \frac{1}{T_1 T_3}$
S^2	$\frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{T_3}$	$\frac{2}{T_1 T_2 T_3}$
S^1	$\dfrac{\left(\frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{T_3}\right)\left(\frac{1}{T_1 T_2} + \frac{1}{T_2 T_3} + \frac{1}{T_1 T_3}\right) - \frac{2}{T_1 T_2 T_3}}{\frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{T_3}}$	0
S^0	$\frac{2}{T_1 T_2 T_3}$	

$\therefore$ if system is stable, then the first column should be positive, which is decided by T_1, T_2, T_3.

$\therefore$ Correct answer is (4).

Problem 2:

1). System transfer function: $H(s) = \dfrac{A\,G(s)}{1+A\,G(s)}$

$$H(s) = \frac{\frac{A}{K}}{\left(\frac{s}{\omega_n}\right)^2 + 2\xi\frac{s}{\omega_n} + 1 + \frac{A}{K}} = \frac{\frac{A}{K}\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2\left(1+\frac{A}{K}\right)}$$

$$= \frac{\frac{A}{K}\omega_n^2\sqrt{\left(1+\frac{A}{K}-\xi^2\right)}}{(s+\xi\omega_n)^2 + \omega_n^2\cdot\left(1+\frac{A}{K}-\xi^2\right)} \cdot \frac{1}{\sqrt{1+\frac{A}{K}-\xi^2}}$$

$\because$ $$\mathcal{L}(e^{-at}\sin\omega t) = \frac{\omega}{(s+a)^2+\omega^2}$$

$r(t) = \delta(t)$ $\therefore$ $R(s) = 1$

$Y(s) = H(s)\cdot R(s) = H(s)$

$\therefore$ $$y(t) = \frac{\frac{A}{K}\omega_n}{\sqrt{1+\frac{A}{K}-\xi^2}}\, e^{-\xi\omega_n t}\sin\left(\omega_n\sqrt{1+\frac{A}{K}-\xi^2}\; t\right)$$

$\therefore$ frequency of oscillation of the output $y(t)$ will change when A changes.

2). A will affect frequency of oscillation of the output, A will also affect the magnitude.

However A won't affect the term $e^{-\xi\omega_n t}$

$\therefore$ A won't change the time required for the oscillatory impulse response to decay away.

Problem 3.

$$G(s) = \frac{1}{s(s+3)}, \quad G_c(s) = \frac{ks}{s+p}$$

transfer function $H(s) = \dfrac{G(s)G_c(s)}{1+G(s)G_c(s)} = \dfrac{ks}{s(s+3)(s+p)+ks}$

$$H(s) = \frac{k}{(s+3)(s+p)+k} = \frac{k}{s^2+(3+p)s+3p+k}$$

$$\begin{cases} 2\xi w_n = 3+p \\ w_n^2 = 3p+k \end{cases}$$

$M_p = 5\%$. $t_s = \frac{3}{4}$ s.

$\because$ settling time for 5% criterion is $\frac{3}{\sigma}$

$\therefore \frac{3}{\sigma} = \frac{3}{4}$ $\quad \sigma = 4$

$\because \xi w_n = \sigma \quad \therefore \xi w_n = 4$.

From M_p versus ξ for the second-order system figure. we know.

$M_p = 5\% \Rightarrow \xi = 0.7$

$\therefore \xi w_n = 0.7 w_n = 4 \quad \therefore w_n = \frac{40}{7}$

$\because 2\xi w_n = 3+p = 8 \quad \therefore p = 5$.

$3p+k = w_n^2 = \frac{1600}{49} = 15+k$

$\therefore k = \frac{1600}{49} - 15 = \frac{865}{49}$

Problem 4.

system is: $\dfrac{as+b}{s^4+as^3+bs^2+cs+d}$

Proof:

the characteristic equation is:

$$s^4+as^3+bs^2+cs+d=0$$

Routh's table.

s^4	1	b	d
s^3	a	c	
s^2	$\dfrac{ab-c}{a}$	d	
s^1	$\dfrac{\frac{abc-c^2}{a}-ad}{\frac{ab-c}{a}}$		
s^0	d		

[illegible] $\because$ a, b, c, d are positive. and $abc-c^2-a^2d>0$

$$\therefore \quad abc-c^2>a^2d>0$$

$$\therefore \quad (ab-c)c>0 \Rightarrow ab-c>0$$

$$\therefore \quad \frac{\frac{abc-c^2}{a}-ad}{\frac{ab-c}{a}}>0.$$

all the numbers in the first column are positive

$\therefore$ System is stable.

EOP

Problem 5.

(1) From figure, we know, open-loop transfer function is:

$$kG(s) = \frac{25\left(\frac{s}{10}+1\right)^4}{s(s+1)\left(\frac{s}{1000}+1\right)^3}$$

$$28 = 20\log k \quad \Rightarrow \quad k = 25$$

(2) closed-loop transfer function: (unity feedback)

$$H(s) = \frac{kG(s)}{1+kG(s)}$$

$$\therefore \quad Y(s) = H(s)\cdot R(s)$$

$$E(s) = R(s) - Y(s) = \frac{1}{1+kG(s)}R(s) = \frac{s(s+1)\left(\frac{s}{1000}+1\right)^3}{s(s+1)\left(\frac{s}{1000}+1\right)^3+25\left(\frac{s}{10}+1\right)^4}R$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\cdot\frac{s(s+1)\left(\frac{s}{1000}+1\right)^3}{s(s+1)\left(\frac{s}{1000}+1\right)^3+25\left(\frac{s}{10}+1\right)^4}\cdot\frac{1}{s^{k+1}}$$

$k=0 \qquad e_{ss}=0$

$k=1 \qquad e_{ss} = \frac{1}{25} = \frac{1}{k_v}$

$\therefore$ System type is 1, error constant $k_v = 25$.

Problem 6.

$$G(s) = \frac{2as+a^2}{s^2}$$

transfer fuction of close-loop: $H(s) = \frac{G(s)}{1+G(s)}$

$$Y(s) = H(s)R(s) = \frac{G(s)}{1+G(s)} R(s)$$

$$E(s) = R(s) - Y(s) = \frac{1}{1+G(s)} R(s)$$

for unit-step input: $R(s) = \frac{1}{s}$

$$\therefore E(s) = \frac{1}{1+\frac{2as+a^2}{s^2}} \cdot \frac{1}{s} = \frac{s}{s^2+2as+a^2} = \frac{s}{(s+a)^2}$$

$$\therefore e(t) = (1-at)e^{-at}$$

$$e^2(t) = (1-at)^2 e^{-2at} = e^{-2at} - 2at\, e^{-2at} + a^2t^2 e^{-2at}$$

$$\mathcal{L}(e^{-2at}) = \frac{1}{s+2a}$$

$$\mathcal{L}(-2at\, e^{-2at}) = -2a \cdot \frac{1}{(s+2a)^2}$$

$$\mathcal{L}(a^2t^2e^{-2at}) = 2a^2 \frac{1}{(s+2a)^3}$$

$$\therefore \mathcal{L}(e^2(t)) = \frac{1}{s+2a} - \frac{2a}{(s+2a)^2} + \frac{2a^2}{(s+2a)^3}$$

$$J_1 = \int_0^{+\infty} e^2(t)dt = \lim_{t\to\infty} \int_0^t e^2(t)\,dt = \lim_{s\to 0} s \cdot \frac{\mathcal{L}(e^2(t))}{s} = \lim_{s\to 0} \mathcal{L}(e^2(t))$$

$$= \lim_{s\to 0} \left[\frac{1}{s+2a} - \frac{2a}{(s+2a)^2} + \frac{2a^2}{(s+2a)^3}\right]$$

$$= \frac{1}{2a} - \frac{2a}{4a^2} + \frac{2a^2}{8a^3}$$

$$= \frac{1}{4a}$$

$$J_2 = \int_0^{+\infty} t e^2(t)\, dt$$

$$= \lim_{t\to\infty} \int_0^t t e^2(t) dt = \lim_{s\to 0} s \cdot \frac{\mathcal{L}(e^2(t)\cdot t)}{s} = \lim_{s\to 0} s \frac{\mathcal{L}(e^2(t)\cdot t)}{s}$$

$$= \lim_{s\to 0} \mathcal{L}(te^2(t))$$

$$\mathcal{L}(te^2(t)) = -\frac{d\mathcal{L}(e^2(t))}{ds}$$

$$= -\left(-\frac{1}{(s+2a)^2} + \frac{4a}{(s+2a)^3} - \frac{6a^2}{(s+2a)^4}\right)$$

$$J_2 = -\lim_{s\to 0}\left(-\frac{1}{(s+2a)^2} + \frac{4a}{(s+2a)^3} - \frac{6a^2}{(s+2a)^4}\right)$$

$$= -\left(-\frac{1}{4a^2} + \frac{4a}{8a^3} - \frac{6a^2}{16a^4}\right)$$

$$= -\left(-\frac{1}{4a^2} + \frac{1}{2a^2} - \frac{3}{8a^2}\right)$$

$$= +\frac{1}{8a^2}$$